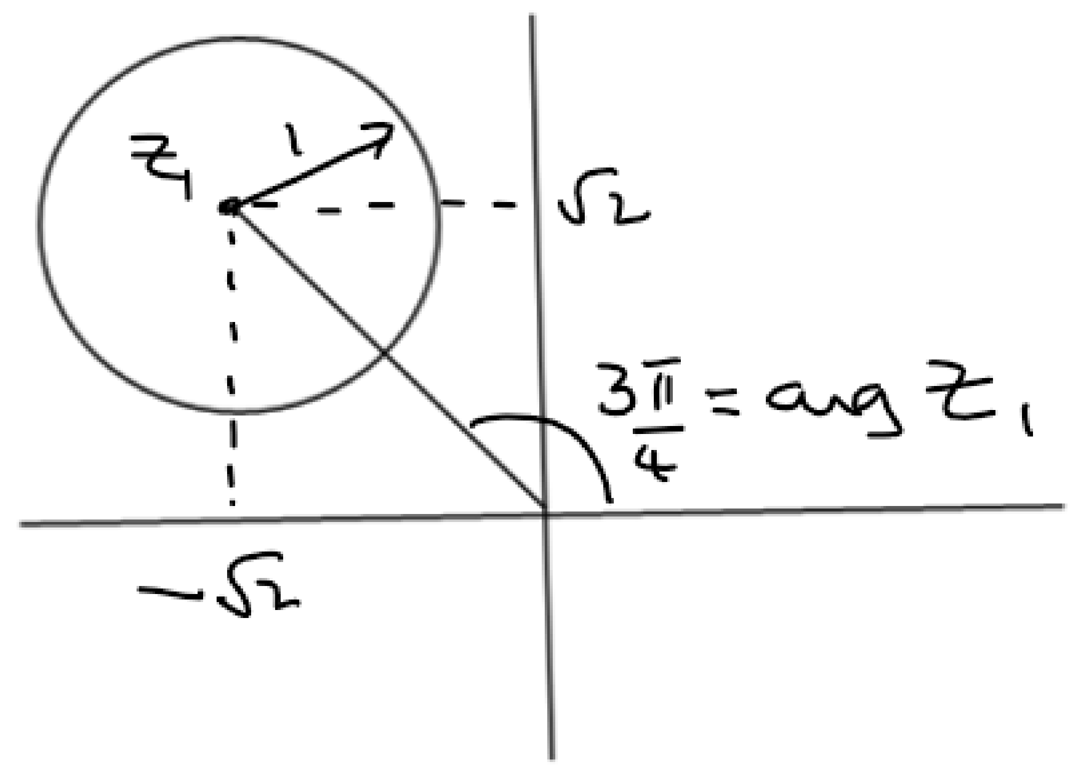
1 (a) (i) $z_1 = 2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$= 2\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$= -\sqrt{2} + i\sqrt{2}$

coordinates of $z_1 = (-\sqrt{2}, \sqrt{2})$

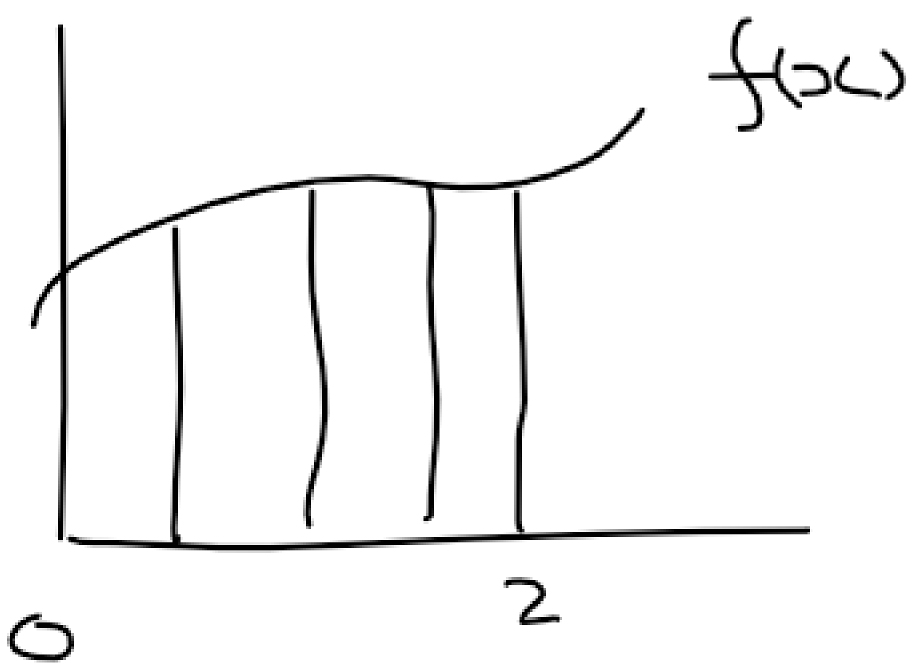
(ii)



(iii) for $|-2+i\sqrt{2} - z_2| = 1$

$\Rightarrow$ circle with centre $(-\sqrt{2}, \sqrt{2})$
radius 1 unit.

b)



x	fx	
0	2	
½		2.031
1		2.236
1.5		2.716
2	3.464	
Σ	5.464	6.983

$\int_0^2 \sqrt{4+x^3}\, dx$

$= \frac{1}{2} \times \frac{1}{2}\left(5.464 + 2(6.983)\right)$

$= 4.8575$

1(c) $\displaystyle\int \frac{\sin^{-1}\left(\frac{x}{2}\right)}{\sqrt{4-x^2}}\,dx$

let $u = \sin^{-1}\left(\frac{x}{2}\right)$

$$\frac{du}{dx} = \frac{1}{2\sqrt{1-\frac{x^2}{4}}}$$

substituting integral becomes

$$\int \frac{u}{2\sqrt{1-\frac{x^2}{4}}}\, 2\sqrt{1-\frac{x^2}{4}}\,du$$

$$= \int u\,du = \frac{u^2}{2} + c$$

$$= \frac{\left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2}{2} + c$$

c(ii) $\frac{1}{2}\left(\sin^{-1}\left(\frac{1}{2}\right)\right)^2 - \frac{1}{2}\left(\sin^{-1} 0\right)^2$

$$= \frac{1}{2}\left(\frac{\pi}{6}\right)^2 - 0 = \frac{\pi^2}{72}$$

2(a)(i) $f(x) = e^x \cos x$ $\quad f(\pi/2) = 0$

$f'(x) = e^x(-\sin x) + e^x \cos x, \; f'\left(\frac{\pi}{2}\right) = -e^{\pi/2}$

$f''(x) = -e^x \cos x - e^x \sin x + -e^x \sin x + e^x \cos x$

$= -2e^x \sin x \quad , \; f''\left(\frac{\pi}{2}\right) = -2e^{\pi/2}$

$$f(x) = -e^{\pi/2}\left(x - \frac{\pi}{2}\right) - e^{\pi/2}\left(x - \frac{\pi}{2}\right)^2$$

(ii) $f(\pi/6) = -e^{\pi/2}\left(-\frac{\pi}{3}\right) - e^{\pi/2}\left(-\frac{\pi}{3}\right)^2$

$= -e^{\pi/2}\left(\frac{\pi^2}{9} - \frac{\pi}{3}\right)$

2 (b) $u_{20} = a + 19d = 35$

$S_{19} = 19a + 171d = 285$

$\Rightarrow a = -3 \quad d = 2$

$S_5 = \frac{5}{2}(-6+8) = 5$

(c) $u_n = ar^{n-1}$

$u_{n+1} = ar^n$

$\frac{u_{n+1}}{u_n} = \frac{ar^n}{ar^{n-1}} = r = \frac{3n+1}{n+2} = \frac{n+2}{n-4}$

$(3n+1)(n-4) = (n+2)^2$

$3n^2 - 11n - 4 = n^2 + 4n + 4$

$2n^2 - 15n - 8 = 0$

$(2n+1)(n-8) = 0$

$n = -\frac{1}{2} \quad n = 8$

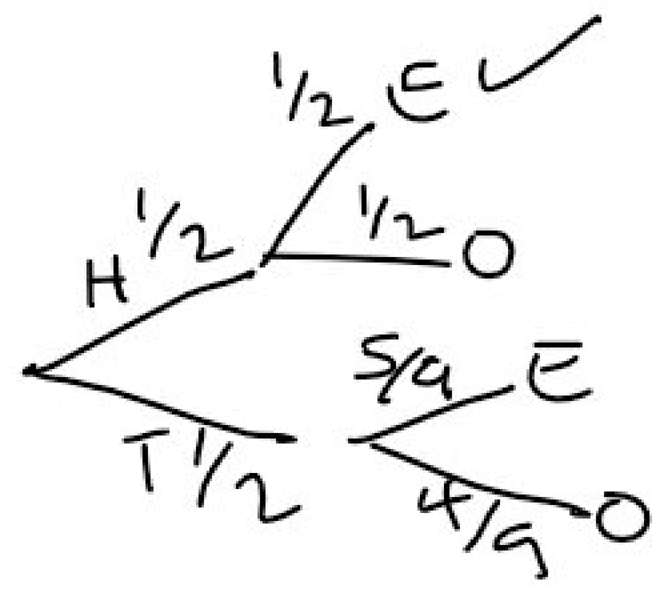
$r = \frac{\frac{3}{2}}{-\frac{9}{2}}$ or $r = \frac{10}{4}$ (not a solution)

$r = -\frac{3}{9} = -\frac{1}{3}$

3 (a)(i)

			DIE 1					
			1	2	3	4	5	6
HEAD	DIE 2	1	2	3	4	5	6	7
		2	3	4	5	6	7	8
		3	4	5	6	7	8	9
		4	5	6	7	8	9	10
		5	6	7	8	9	10	11
		6	7	8	9	10	11	12
TAIL		1	2	3	4			
		2	3	4	5			
		3	4	5	6			

(ii) $P(\text{sum is even}) = \dfrac{23}{45}$ //



(iii) $P(\text{Head AND even}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ //

(iv) If P(H) and P(E) are independent

the $P(H \cap E) = P(H) \times P(E)$

$\frac{1}{4} \neq \frac{1}{2} \cdot \frac{23}{45}$

So not independent

3(b) i $\quad y' + y = 2\sin x$

$I \cdot F = e^{x}$

$$ye^{x} = \int 2e^{x}\sin x \, dx = 2\int e^{x}\sin x \, dx$$

$$e^{x} = 2\left[e^{x}\int \sin x \, dx - \int e^{x}\int \sin x \, dx\right]$$

$$= 2\left[e^{x}(-\cos x) - \int e^{x}(-\cos x)\, dx\right.$$

$$2\int e^{x}\sin x = 2\left[e^{x}(-\cos x) + \int e^{x}\cos x \, dx\right]$$

$$= 2\left(e^{x}(-\cos x) + \left[e^{x}\sin x - \int e^{x}\sin x \, dx\right.\right.$$

$$\int e^{x}\sin x \, dx = e^{x}\sin x - e^{x}\cos x - \int e^{x}\sin x \, dx$$

$$2\int e^{x}\sin x = e^{x}(\sin x - \cos x) + A$$

so

$$ye^{x} = e^{x}(\sin x - \cos x) + A$$

$$\boxed{y = \sin x - \cos x + Ae^{-x}}$$

(ii) when $x = 0 \quad y = 1$

$$1 = \sin 0 - \cos 0 + A$$

so Particular solution

$$\boxed{y = \sin x - \cos x + 2e^{-x}}$$