C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2 – Paper 032

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.

2. Each section consists of ONE question.

3. Answer ALL questions.

4. Write your answers in the spaces provided in this booklet.

5. Do NOT write in the margins.

6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.

7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**

8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

<u>**Examination Materials Permitted**</u>

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

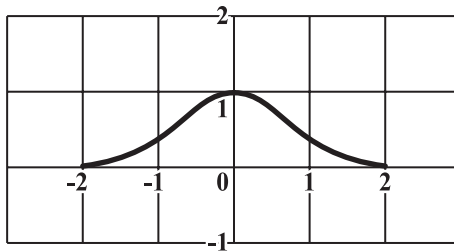"*"Barcode Area"*"
Sequential Bar Code

SECTION A

Module 1

Answer this question.

1. (a) The graph below (**not drawn to scale**) shows the function $f(x) = e^{-x^2}$.

Use the trapezium rule with 5 ordinates (4 divisions) to estimate $\int_{-1}^{0} e^{-x^2} dx$.

[4 marks]

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(b)   A complex number, $z = x + iy$ is given as $z = 8 - 6i$.

Given that $\text{Re}(q^2) = x^2 - y^2$ and $\text{Im}(q^2) = 2xy$, determine q such that $q = \sqrt{z}$.

Note that Re(q) and Im(q) are the real and imaginary parts, respectively.

[6 marks]

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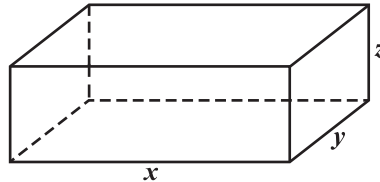
(c) The diagram below (**not drawn to scale**) shows a rectangular cardboard box without a lid.

The box is made from 27m^2 of material. The length, width and height of the box are x, y and z, respectively, and it is known that $z = \dfrac{27 - xy}{2(x + y)}$.

Given that the volume, V, of the box is $V = \dfrac{27xy - x^2y^2}{2(x + y)}$ and that V is maximized when $\dfrac{\partial V}{\partial x} = 0$, $\dfrac{\partial V}{\partial y} = 0$, $x > 0$ and $y > 0$, determine the height, z, of the box that maximizes its volume.

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[10 marks]

Total 20 marks

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SECTION B

Module 2

Answer this question.

2. (a) (i) Express the number $0.\overline{65} = 0.6565656565 \dots$ as a geometric series with the first term $a = \dfrac{65}{100}$.

[3 marks]

(ii) Hence, express $0.\overline{65}$ as a ratio of integers.

[4 marks]

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(b) (i) Obtain the binomial expansion of $\left(1 + \dfrac{1}{3}x\right)^5$ up to and including the **first** THREE non-zero terms.

[4 marks]

(ii) Hence, compute 1.033^5 to 3 decimal places.

[3 marks]

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(c) A function f is defined by $f(x) = 2x \sin x - 3$. Use three iterations of the Newton–Raphson iteration formula and the initial approximation $x_1 = 1$ to obtain a better approximation of the root of $f(x) = 0$.

[6 marks]

Total 20 marks

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SECTION C

Module 3

Answer this question.

3. (a) A farmer has 300 acres of land on which THREE crops, *A*, *B* and *C* are to be cultivated. The costs of cultivating *A*, *B* and *C* are \$30, \$40 and \$50 per acre, respectively, and the farmer has a total of \$11 000 to spend on cultivation.

The hours of labour required for the crops *A*, *B* and *C* are 10, 15 and 40 hours, respectively. A maximum of 6 000 hours of labour are available. The number of acres of land available for crops A, B and C are *x*, *y* and *z*, respectively.

   (i) Write the system of linear equations to represent the number of acres of each crop that the farmer can cultivate.

[3 marks]

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(ii)   By converting the equations into matrix form and reducing it to echelon form, determine how many acres per crop the farmer may cultivate.

[9 marks]

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(b)   A committee of six individuals is to be formed from seven women and three men.

    (i)   In how many ways can the committee be formed with at least one man included?

[4 marks]

    (ii)   What is the probability that such a committee will include five women?

[4 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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