



## CARIBBEAN EXAMINATIONS COUNCIL

## CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

## PURE MATHEMATICS

## UNIT 2 – Paper 02

## ANALYSIS, MATRICES AND COMPLEX NUMBERS

*2 hours 30 minutes***READ THE FOLLOWING INSTRUCTIONS CAREFULLY.**

1. This examination paper consists of **THREE** sections.
2. Each section consists of **TWO** questions.
3. Answer **ALL** questions from the **THREE** sections.
4. Write your answers in the spaces provided in this booklet.
5. Do **NOT** write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. If you use the extra page(s), you **MUST** write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.

**Examination Materials Permitted**

Mathematical formulae and tables (provided) – Revised 2012

Mathematical instruments

Silent, non-programmable electronic calculator

**DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.**



SECTION A

Module 1

Answer BOTH questions.

1. (a) Given that  $3y^2 - 4xy + \sin xy = 5$ , determine  $\frac{dy}{dx}$ .

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- Each section consists of TWO parts.
- Answer ALL questions from the THREE sections.
- Write your answers in the spaces provided in this booklet.
- Do NOT write in the margins.
- Answers should be written in the question and answer book and must be written in the answer book in the question and answer book.
- If you need to write an answer and there is not enough space in the answer book, you may use the extra paper provided in the back of the booklet. Remember to draw a line through your original answer.
- If you use the extra paper, you will lose the question number which is the one provided at the top of the extra paper and you must indicate the question part beside the answer.

[6 marks]

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(b) Differentiate  $\cos^{-1}(3x - 2)$ , expressing your answer in its simplest form.

[3 marks]

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0 2 2 3 4 0 2 0 0 5



- (c) (i) Use partial fractions to show that  $\frac{2x+1}{3x^2-x-2} = \frac{3}{5(x-1)} + \frac{1}{5(3x+2)}$ .

[7 marks]

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(ii) Hence, or otherwise, determine  $\int \frac{2x+1}{3x^2-x-2} dx$ .

[4 marks]

(d) Given that  $x = 2t - \sin t$  and  $y = 1 - \cos t$ , determine  $\frac{d^2y}{dx^2}$ .

[5 marks]

Total 25 marks

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2. (a) Determine  $\int e^{2x} \sin 3x \, dx$ .

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[8 marks]

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- (b) Use DeMivre's theorem to show that  $(1 + 3i)^4 = 28 - 96i$ .

[7 marks]

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- (c) (i) Derive the reduction formula

$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx.$$

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[6 marks]





(ii) Hence, or otherwise, show that

$$\int \sin^6 x \, dx = -\frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x + \frac{5}{8} \int \sin^2 x \, dx.$$

[4 marks]

**Total 25 marks**





**SECTION B**

**Module 2**

**Answer BOTH questions.**

3. (a) Use two iterations of the Newton–Raphson method, with the initial estimate  $x_0 = 0.3$ , to calculate a new estimate for the root of  $f(x) = 3x^3 - 4x + 1$ .

**[6 marks]**

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(b) Determine the Taylor series expansion of  $f(x) = \frac{1}{1-2x}$  about  $x = 3$  up to and including the term in  $x^2$ .

[9 marks]

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- (c) (i) Obtain the binomial expansion of  $(16 - 5x)^{\frac{1}{4}}$  up to and including the term in  $x^2$ .

[6 marks]

- (ii) Hence, state the values for which the expansion of  $(16 - 5x)^{\frac{1}{4}}$  is valid.

[2 marks]

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- (iii) Use the expansion obtained in (c) (i) to estimate the value of  $(16 - 5x)^{\frac{1}{4}}$  when  $x = 3$ .

[2 marks]

Total 25 marks

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4. (a) (i) Use the intermediate value theorem to prove that  $f(x) = 2x^3 + x^2 + 9x - 5$  has at least one root in the interval  $[0.2, 1]$ .

[3 marks]

- (ii) Hence, use three iterations of the interval bisection method to estimate the root of  $f$  in the interval  $[0.2, 1]$ .

[5 marks]





(b) The first 5 terms of a sequence are  $4, -\frac{7}{2}, \frac{10}{6}, -\frac{13}{24}, \frac{16}{120}, \dots$

(i) Write down an expression for the  $n^{\text{th}}$  term of the sequence.

[4 marks]

(ii) Determine whether the sequence converges or diverges.

[3 marks]

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(c) Calculate  $\sum_{r=11}^{27} r$ .

[4 marks]

(d) Use the sum of a series to calculate an equivalent fraction for the repeating decimal 0.2727272 ...

[6 marks]

Total 25 marks

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## SECTION C

## Module 3

Answer BOTH questions.

5. (a) Given that the matrix  $A = \begin{bmatrix} 1-p & 3 & -2 \\ 2 & p & -1 \\ -3 & 2 & 1 \end{bmatrix}$  is singular, calculate the value(s) of  $p$ .

[6 marks]

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- (b) Use row reduction to solve the following system of equations.

$$3x - 4y + z = 16$$

$$2x + y - 2z = 5$$

$$x + 2y - z = -2$$

[9 marks]

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(c) A committee consisting of 3 men and 2 women is to be formed from a group of 6 men and 4 women. However, it is known in advance that 2 of the men who dislike each other will not serve together on the committee.

(i) In how many different ways can the committee be formed?

[7 marks]

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(ii) What is the probability that the committee formed will contain neither of the 2 men who refuse to serve together?

[3 marks]

Total 25 marks

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6. (a) A chemical test kit is 95% accurate in detecting when a solution is acidic. However, the test kit also yields a 'false positive' result for 1% of all solutions tested. If 0.5% of the solutions tested are acidic, what is the probability that a solution is acidic given that the test result is positive?

[5 marks]

- (b) Determine the general solution of the differential equation  $\frac{dy}{dx} + \frac{3}{x}y = \frac{\sin 2x}{x^3}$ .

[5 marks]

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(c) A differential equation is given as  $4y'' - 6y' + 7y = 0$ .

(i) Show that the general solution of the differential equation is

$$Ae^{\frac{3}{4}x} \sin\left(\frac{\sqrt{19x}}{4}\right) + Be^{\frac{3}{4}x} \cos\left(\frac{\sqrt{19x}}{4}\right), \text{ where } A \text{ and } B \text{ are constants.}$$

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**[6 marks]**

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- (ii) Hence, or otherwise, determine the particular solution of the differential equation given that at  $x = 0$ ,  $y = 3$  and  $y' = 0$ .



Question No.

[9 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.