CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.

2. Each section consists of TWO questions.

3. Answer ALL questions from the THREE sections.

4. Write your answers in the spaces provided in this booklet.

5. Do NOT write in the margins.

6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.

7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**

8. **If you use the extra page(s), you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – Revised 2012
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

Module 1

Answer BOTH questions.

1. (a) (i) Let **p** and **q** be any two propositions. Complete the truth table below.

p	q	p $\longrightarrow$ q	q $\longrightarrow$ p	(p $\longrightarrow$ q) $\wedge$ (q $\longrightarrow$ p)
T	T			
T	F			
F	T			
F	F			

[3 marks]

(ii) Hence, state whether the statements **q $\longrightarrow$ p** and **(p $\longrightarrow$ q) $\wedge$ (q $\longrightarrow$ p)** are logically equivalent. **Justify your response.**

....

....

....

[2 marks]

(b) Let x and y be negative real numbers and let z be any real number. Use a counter example to show that the statement "if $x > y$ then $xz > yz$" is false.

[3 marks]

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(c)   The expression $f(x) = 6x^3 + px^2 + qx + 2$ is divisible by $2x - 1$ and has a remainder of 2 when divided by $x - 1$.

Calculate the values of p and q.

[9 marks]

(d)   (i)   Solve the logarithmic equation $\log_3 (x^2 - 9) - \log_3 (x + 3) = 3$.

<div align="right">[4 marks]</div>

(ii)   Show that $\sqrt{320x^3} + \sqrt{125x^3}$ simplifies to $13x\sqrt{5x}$.

<div align="right">[4 marks]</div>

<div align="right">Total 25 marks</div>

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2.   (a)   Let $f(x) = 7x + 2$. Prove that f is bijective.

[5 marks]

(b) The roots of the cubic equation $3x^3 - x^2 - 2x + 1 = 0$ are α, β and γ. Determine the equation whose roots are $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$ and $\dfrac{1}{\gamma}$.

[8 marks]

(c)   The diagram below shows the graph of the curve $f(x) = x^2 + 6x + 8$.
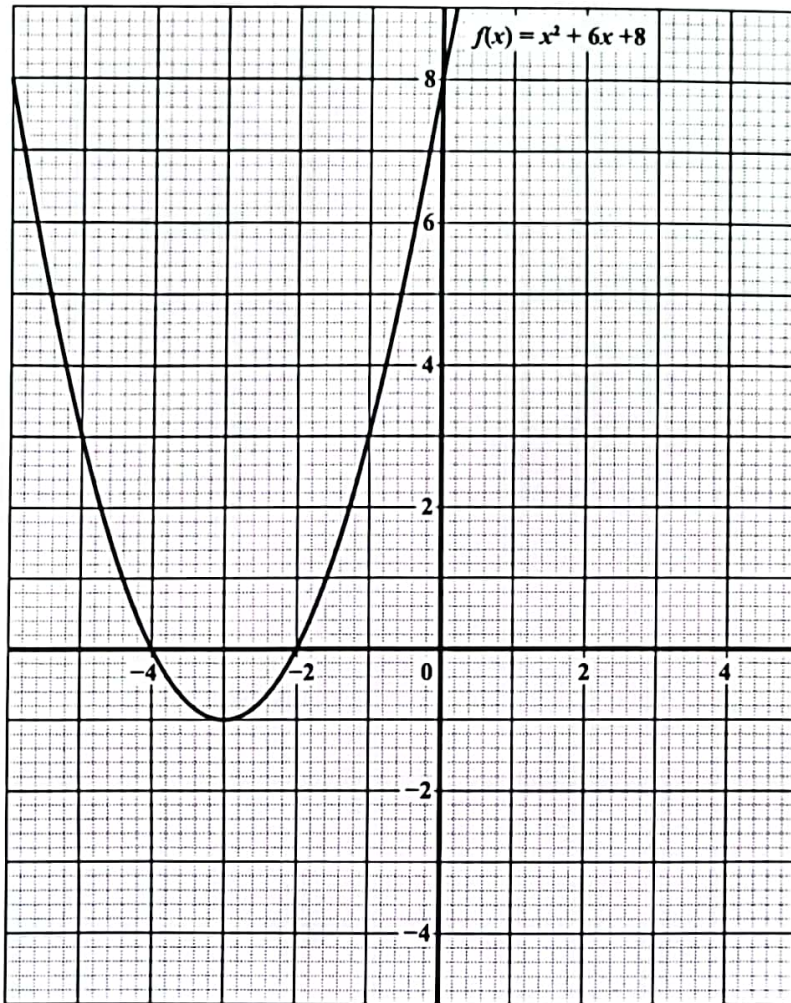
$f(x) = x^2 + 6x + 8$

(i)   On the axes provided above, sketch and label the graph of $g(x) = |x^2 + 6x + 8|$.

[3 marks]

(ii)   On the same axes, sketch and label the inverse of f for $x \geq -3$.   [5 marks]

(d)   Given that $g(x) = \dfrac{2x + 3}{x + 3}$, prove that $g^{-1}(2)$ does not exist.

[4 marks]

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3.   (a)   Prove that

$$\frac{1 - \sin \theta}{1 + \sin \theta} = (\sec \theta - \tan \theta)^2.$$

[9 marks]

(b)   Solve the equation $2 \cos^2 x - 3 \sin x = 3$ for $0 \le x \le 2\pi$.

[9 marks]

0 2 1 3 4 0 2 0 1 2

(c) Show that $\cos\left(\dfrac{\pi}{2} + x\right) = -\sin x$.

[4 marks]

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(d) A and B are acute angles such that $\sin A = \dfrac{3}{5}$ and $\cos B = \dfrac{5}{13}$. Calculate, without using

tables or calculators, the EXACT value of $\cos (A - B)$.

[3 marks]

Total 25 marks

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4. (a) Obtain the Cartesian equation of the curve given in parametric form

$x = 3 \cos t$ and $y = 4 \sin t$.

[5 marks]

(b) The equation of a line is $x = 2 + t$, $y = 1 - 3t$ and $z = 4 + t$, and the equation of a plane is $x + 2y + z = 12$. Determine the point of intersection of the line and the plane.

[6 marks]

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(c)   (i)   Determine the vector equation of the plane which passes through $(1, 5, -1)$ and

which is perpendicular to the vector $\begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$.

[4 marks]

(ii)   Hence, determine the coordinates of the point in the plane where $y = 3$ and $z = 1$.

[3 marks]

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(d) Given that a line is parallel to the vector $u = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ and that the vector $v = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ is normal to the plane, calculate the angle between the line and the plane.

[7 marks]

Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5.   (a)   Determine $\displaystyle\lim_{x \to \infty} \frac{2x^3 - 4x + 1}{3x^4 + x^2 - 2}$.

[5 marks]

0 2 1 3 4 0 2 0 1 8

(b)   A student goes to the water cooler to get a drink of water. The student has a cup in the shape of a cone. The water from the cooler is filling the cup at the rate of 12 cm³ per second. If the height, h, of the cup is 16 cm and the radius, r, of its circular opening is 8 cm, how fast is the water in the cup rising when the height is 4 cm?

The volume of a cone is $V = \dfrac{1}{3}\pi r^2 h$, where r is the radius and h is the height.
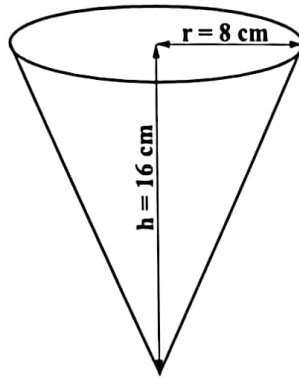
[8 marks]

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(c) A manufacturer of containers wants to make a container in the shape of a cuboid with an open top. The container is made from a flat sheet of metal of length 16 metres and width 13 metres. Squares of length x metres are cut from each corner of the sheet to create the sides of the container.

(i) Show that the volume of the container is $4x^3 - 58x^2 + 208x$.

[2 marks]

(ii)   Using the method of the second derivative, determine the height, x, that will maximize the volume of the container.

[9 marks]

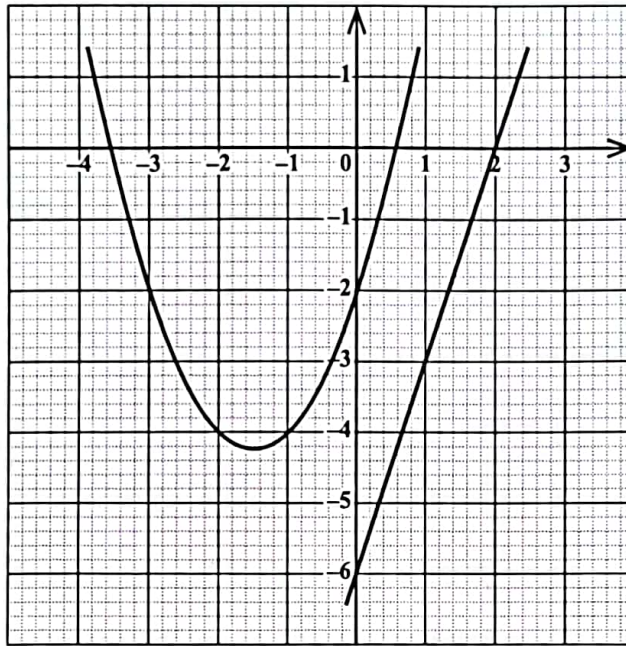(iii)   Determine the maximum volume of the container.

[1 mark]

Total 25 marks

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6. (a) Calculate the volume of the solid generated by revolving the region bounded by the line $y = 3x - 6$ and the parabola $y = x^2 + 3x - 2$, on the interval $[0, 1]$ about the x-axis.
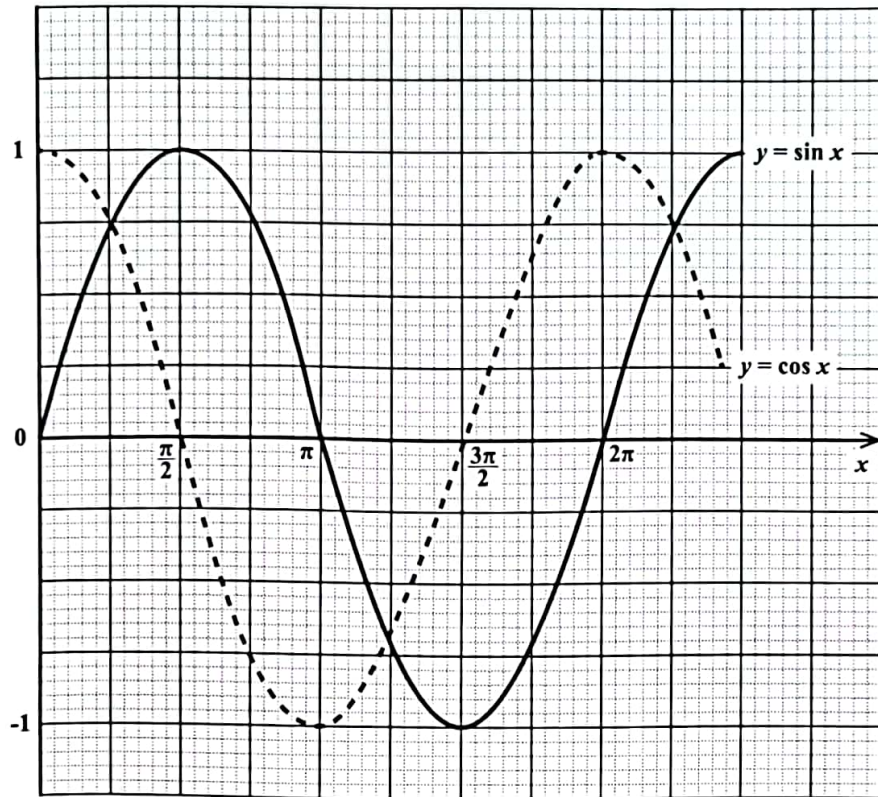
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[7 marks]

(b) The diagram below shows the curves $y = \cos x$ and $y = \sin x$. Determine the area bounded by the curves between $x = \dfrac{\pi}{4}$ and $x = \dfrac{3\pi}{2}$.

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[7 marks]

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(c) On the surface of the moon, the acceleration due to gravity is -1.625 m/s^2. A rocket is launched away from the moon with an initial height of 1 200 metres and an initial velocity of 30 m/s.

Determine the velocity and height of the rocket 5 seconds after launch.

[7 marks]

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(d)   Determine $\int \cos^3 2x \sin 2x \, dx$.

[4 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.

0 2 1 3 4 0 2 0 2 8