FORM TP 2022327

TEST CODE 02134020

MAY/JUNE 2022

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of TWO questions.
3. Answer ALL questions from the THREE sections.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

DO NOT
WRITE ON
THIS PAGE

SECTION A

Module 1

Answer BOTH questions.

1. (a) (i) Let **p** and **q** be any two propositions. Complete the truth table below.

p	**q**	**~q**	**(~q ∧ p)**	**p ∨ (~q ∧ p)**
T	T			
T	F			
F	T			
F	F			

[3 marks]

(ii) Hence, state whether the statements **(~q ∧ p)** and **p ∨ (~q ∧ p)** are logically equivalent. **Justify your response.**

.....

.....

[2 marks]

(b) Determine the values of a, b, and c, such that

$$2x^2 - 7x + 12 = a(x - 2)(x - 1) - b(x - 3) + c.$$

[9 marks]

(c) Solve the inequality $|3x + 2| \geq 4$.

[5 marks]

(d) Solve the logarithmic equation $\log_5 (x + 2) + \log_5 (x + 6) = 1$.

[6 marks]

Total 25 marks

2. (a) Let $f(x) = \dfrac{3x + 1}{x}$ and $g(x) = e^{-2x} + 1$.

(i) Show that $f^{-1}(x) = \dfrac{1}{x - 3}$.

[3 marks]

(ii) Hence, or otherwise, write an expression for $(f^{-1} \circ g)(x)$.

[2 marks]

(b) Solve the equation $6 - \frac{7}{2^{2x}} - \frac{3}{4^{2x}} = 0$.

[8 marks]

(c) The function $f(x) = 2x^3 - px^2 + qx - 10$ is divisible by $x - 1$ and has a remainder of -6 when divided by $x + 1$.

Calculate the values of p and q.

[6 marks]

(d) The roots of the equation $2x^3 - x^2 + 3x - 1 = 0$ are α, β, and γ.

Given that $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{3}{2}$ and $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma} = \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma}$, determine the equation whose roots are $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$.

[6 marks]

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) Prove that

$$\frac{\tan\theta \sin\theta}{1-\cos\theta} = 1 + \frac{1}{\cos\theta}.$$

[6 marks]

(b) Solve the equation $\tan^2 \theta - 2 \tan \theta = 3$ for $0 \leq \theta \leq 2\pi$.

[6 marks]

(c) (i) Show that $4 \cos \theta + 3 \sin \theta = 5 \sin (\theta + 0.927°)$.

[5 marks]

(ii) Hence, or otherwise, solve the equation $4 \cos \theta + 3 \sin \theta = 0$.

[4 marks]

(d) Given that $\sin A = \frac{1}{3}$ and A is obtuse, calculate the value of $\cos A$ without using a calculator.

[4 marks]

Total 25 marks

4. (a) A circle has equation $x^2 + y^2 - 10x + 4y - 5 = 0$.

(i) Determine the centre and radius of the circle.

[4 marks]

(ii) Determine the equation of the tangent to the circle at the point (2, 3).

[4 marks]

(b) (i) Determine the equation of the plane that passes through the point (0, 2, –2) and which is perpendicular to the vector $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$.

[3 marks]

(ii) Determine the angle between the vector $3\mathbf{i} - 2\mathbf{j}$ and the x-axis.

[3 marks]

(c) The parametric equations of a line are given as

$$x = 2 + 5\lambda,\ y = -3 + 4\lambda,\ z = 4 - 2\lambda.$$

Determine the coordinates of the point where the line crosses the xy plane.

[5 marks]

(d) A point $P(x, y)$ moves in the xy plane such that it is the same distance from the point $A(1, 2)$ as it is from the line $x = 3$. Determine the equation of the locus of P.

[6 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION C

Module 3

Answer BOTH questions.

5. (a) Determine $\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 + x - 6}$.

[5 marks]

(b) A ladder which is 10 metres long is leaning against a wall. The bottom of the ladder is sliding away from the base of the wall at a rate of 4 m/s. Determine the rate at which the top of the ladder is moving down the wall when the bottom of the ladder is 6 metres from the base of the wall.

[7 marks]

(c) A function f is given as $f(x) = 4x^3 - 3x^2 + 1$, for $-1 \leq x \leq 1$.

(i) Determine the coordinates of the stationary points of the function f.

[6 marks]

(ii) Determine the nature of these stationary points.

[4 marks]

(iii) Determine the **absolute** maximum and minimum values of the function f.

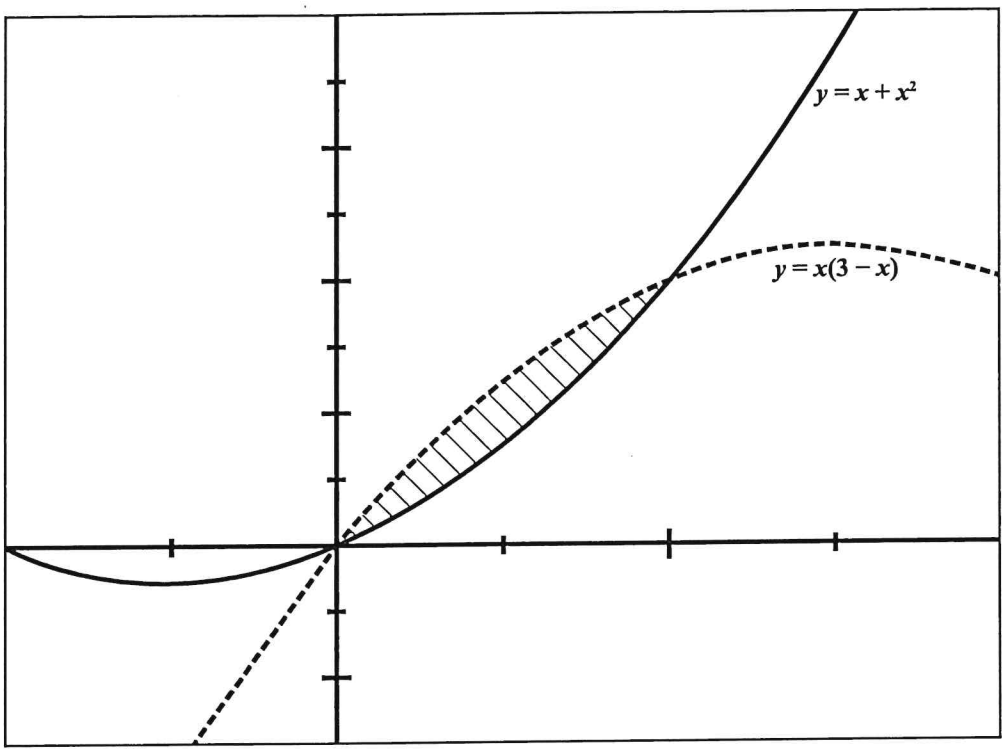
[3 marks]

Total 25 marks

6. (a) Using the substitution $u = x^2 + 1$, determine $\int 4x\,(x^2 + 1)^5\,dx$.

[5 marks]

(b) The diagram below shows two curves, $y = x + x^2$ and $y = x\,(3 - x)$. Calculate the area of the shaded region.



[6 marks]

(c) Calculate the volume of the solid generated by revolving the region bounded by the line $y = 6x$ and the parabola $y = 6x^2$ about the x-axis.

[6 marks]

(d) Solve the differential equation $\frac{dy}{dx} = \frac{x + 4x^2}{y^2}$, given that $y = 6$ when $x = 0$.

[8 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.