CARIBBEAN EXAMINATIONS COUNCIL CARIBBEAN ADVANCED PROFICIENCY EXAMINATION ${ }^{\circledR}$

FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

TEST CODE

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SUBJECT PURE MATHEMATICS - UNIT 2 - Paper 032

PROFICIENCY ADVANCED

REGISTRATION NUMBER


| NAME OF SCHOOL/CENTRE |
| :---: |
|  |


| CANDIDATE'S FULL NAME (FIRST, MIDDLE, LAST) |
| :---: |
|  |

DATE OF BIRTH


SIGNATURE $\qquad$

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| :---: |
| Sequential Bar Code |



# CARIBBEAN EXAMINATIONS COUNCIL <br> CARIBBEAN ADVANCED PROFICIENCY EXAMINATION ${ }^{\circledR}$ <br> PURE MATHEMATICS 

UNIT 2 - Paper 032

## ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 hour 30 minutes

## READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of ONE question.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. Remember to draw a line through your original answer.
8. If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.

## Examination Materials Permitted

Mathematical formulae and tables (provided) - Revised 2012
Mathematical instruments
Silent, non-programmable electronic calculator
do Not TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
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## SECTION A

## Module 1

## Answer this question.

1. (a) A rectangular box has dimensions, $x, y, z$ each of which is measured with a maximum error of 0.1 cm .

Determine the maximum error of the volume, $V$, of the box given that $x=75 \mathrm{~cm}$, $y=60 \mathrm{~cm}, z=40 \mathrm{~cm}$ and the maximum error is $d V=\frac{\partial V}{\partial x} d x+\frac{\partial V}{\partial y} d y+\frac{\partial V}{\partial z} d z$.
(b) Determine $z$ such that $z^{2}-(6+2 i) z+(7+6 i)=0$.
(c) Given rectangles with a fixed perimeter, $p$, show that a square has the maximum area.

## SECTION B

## Module 2

## Answer this question.

2. (a) Use mathematical induction to prove that $\frac{1}{2}+\frac{1}{6}+\ldots+\frac{1}{n(n-1)}=1-\frac{1}{n}, n \geq 2$.
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(b) The function $f(x)=5 \ln x-x+2$ has a root in the interval [0,1]. By letting $x_{1}=1$ and using the Newton-Raphson method, show that an approximation of the root of $f$ in the interval $[0,1]$ can be expressed as $\frac{3}{4}-\frac{15}{17}\left(\frac{1}{4}+\ln \frac{3}{4}\right)$.
(c) A series is given as $\sum_{r=3}^{n} \frac{4}{r(r-2)}$.
(i) Show that $\left[\frac{1}{r-2}-\frac{1}{r}\right)=\frac{2}{r(r-2)}$ where $r \in \mathrm{~N}$.
(ii) Hence, or otherwise, determine $\sum_{r=3}^{n} \frac{4}{r(r-2)}$ in terms of $n$.

## SECTION C

## Module 3

## Answer this question.

3. (a) Determine the general solution of the differential equation $\frac{d y}{d x}+(\cos x) y=\cos x$.
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(b) Two fair coins are tossed along with a fair, six-sided die.
(i) List the simple events of the possibility space.
(ii) Determine the probability that at least one head or a number greater than 4 is observed.
(c) $\quad \mathbf{A}$ and $\mathbf{B}$ are two matrices such that $\operatorname{det}(\mathbf{A B})=-880$ and $\operatorname{det}(\mathbf{A})=88$. Determine $x$, given that $\mathbf{B}=\left(\begin{array}{rrr}1 & 3 & -2 \\ 2 & x & 1 \\ 3 & 1 & -1\end{array}\right)$.

## EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided. Question No. $\square$
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If you use this extra page, you MUST write the question number clearly in the box provided.
Question No. $\square$
"*"Barcode Area"
Sequential Bar Code


## CANDIDATE'S RECEIPT

## INSTRUCTIONS TO CANDIDATE:

1. Fill in all the information requested clearly in capital letters.

TEST CODE:

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SUBJECT: PURE MATHEMATICS - UNIT 2 - Paper 032

PROFICIENCY:
ADVANCED

REGISTRATION NUMBER:


FULL NAME: $\qquad$
(BLOCK LETTERS)

Signature: $\qquad$

Date: $\qquad$
2. Ensure that this slip is detached by the Supervisor or Invigilator and given to you when you hand in this booklet.
3. Keep it in a safe place until you have received your results.

## INSTRUCTION TO SUPERVISOR/INVIGILATOR:

Sign the declaration below, detach this slip and hand it to the candidate as his/her receipt for this booklet collected by you.

I hereby acknowledge receipt of the candidate's booklet for the examination stated above.

Signature: $\qquad$
Supervisor/Invigilator

Date: $\qquad$

