CARIBBEAN EXAMINATIONS COUNCIL CARIBBEAN ADVANCED PROFICIENCY EXAMINATION ${ }^{\circledR}$


FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

TEST CODE

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SUBJECT PURE MATHEMATICS - UNIT 2 - Paper 032

PROFICIENCY ADVANCED

REGISTRATION NUMBER


| NAME OF SCHOOL/CENTRE |
| :---: |
|  |


| CANDIDATE'S FULL NAME (FIRST, MIDDLE, LAST) |
| :---: |
|  |

DATE OF BIRTH


SIGNATURE $\qquad$

| "*"Barcode Area"*" |
| :---: |
| Sequential Bar Code |



TEST CODE 02234032

# CARIBBEAN EXAMINATIONS COUNCIL <br> CARIBBEAN ADVANCED PROFICIENCY EXAMINATION ${ }^{\circledR}$ <br> PURE MATHEMATICS 

UNIT 2 - Paper 032

## ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 hour 30 minutes

## READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of ONE question.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. Remember to draw a line through your original answer.
8. If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.

## Examination Materials Permitted

Mathematical formulae and tables (provided) - Revised 2012
Mathematical instruments
Silent, non-programmable electronic calculator
do Not TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
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## SECTION A

## Module 1

## Answer this question.

1. (a) A complex number $z_{1}$ is such that $\left|z_{1}\right|=2$ and $\arg z_{1}=\frac{5 \pi}{6}$.

Use the Argand diagram below to answer (a) (i), (ii) and (iii).

(i) Determine the coordinates of $z_{1}$.
(ii) Connect $z_{1}$ to the Origin with a straight line and label the angle that represents $\arg z_{1}$.
(iii) Sketch the locus of the point $z$ which moves in the complex plane such that $\left|z-z_{1}\right|=1$.
(b) Use the trapezium rule, with four smaller intervals of equal width, to approximate $\int_{0}^{2} \sqrt{3+x^{2}}$.
(c) Determine $\int_{0}^{1} \frac{d x}{\sqrt{9-x^{2}}}$.
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## SECTION B

## Module 2

## Answer this question.

2. (a) (i) Show that $\frac{5}{4 r^{2}-1}=\frac{5}{2(2 r-1)}-\frac{5}{2(2 r+1)}$.
(ii) Hence, or otherwise, show that $\sum_{r=1}^{n} \frac{5}{4 r^{2}-1}=\frac{5 n}{2 n+1}$.
(b) An arithmetic progression is such that the fourth and tenth partial sums are $S_{4}=-24$ and $S_{10}=0$, respectively.
(i) Calculate the first term and the common difference.
(ii) Hence, or otherwise, calculate the 15 th term of the progression.
(c) (i) Show that the equation $2 e^{x}+x^{2}-3=0$ has a root, $\alpha$, in the interval $(-2,-1)$.
(ii) Apply linear interpolation once in the interval $(-2,-1)$ to find an approximation to the root, $\alpha$.

## SECTION C

## Module 3

## Answer this question.

3. (a) (i) To make three-letter codes, three letters are selected without replacement from the word TRAVEL and are written down in the order in which they are selected. How many three-letter codes may be formed?
(ii) For a three-letter code to be legal, it must have at least one vowel. What is the probability that a legal word is formed on a single attempt?
(b) A system of equations $\mathbf{A x}=\mathbf{b}$ is given by

$$
\left(\begin{array}{rrr}
1 & 1 & -1 \\
2 & -1 & 3 \\
1 & -2 & -2
\end{array}\right] \quad\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left(\begin{array}{r}
6 \\
-9 \\
3
\end{array}\right) .
$$

(i) Calculate $|\mathbf{A}|$.
(ii) Let the matrix $\mathbf{C}=\left[\begin{array}{rrr}8 & 7 & -3 \\ 4 & -1 & 3 \\ 2 & -5 & -3\end{array}\right)$. Show that $\mathbf{C}^{T} \mathbf{A}-18 \mathbf{I}=0$.
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(iii) Hence, or otherwise, obtain $\mathbf{A}^{-1}$.
(iv) Solve the given system of equations for $x, y$ and $z$.
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## EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.


#### Abstract

Question No.


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## EXTRA SPACE

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Question No. $\square$
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## CANDIDATE'S RECEIPT

## INSTRUCTIONS TO CANDIDATE:

1. Fill in all the information requested clearly in capital letters.

TEST CODE:

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SUBJECT: PURE MATHEMATICS - UNIT 2 - Paper 032

PROFICIENCY:
ADVANCED

REGISTRATION NUMBER:


FULL NAME: $\qquad$
(BLOCK LETTERS)

Signature: $\qquad$

Date: $\qquad$
2. Ensure that this slip is detached by the Supervisor or Invigilator and given to you when you hand in this booklet.
3. Keep it in a safe place until you have received your results.

## INSTRUCTION TO SUPERVISOR/INVIGILATOR:

Sign the declaration below, detach this slip and hand it to the candidate as his/her receipt for this booklet collected by you.

I hereby acknowledge receipt of the candidate's booklet for the examination stated above.

Signature: $\qquad$
Supervisor/Invigilator

Date: $\qquad$

