C A R I B B E A N E X A M I N A T I O N S C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

"*"Barcode Area"*"
Front Page Bar Code

11 JUNE 2019 (p.m.)

FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

TEST CODE | 0 | 2 | 1 | 3 | 4 | 0 | 3 | 2 |

SUBJECT PURE MATHEMATICS – UNIT 1 – Paper 032

PROFICIENCY ADVANCED

REGISTRATION NUMBER | | | | | | | | | | |

SCHOOL/CENTRE NUMBER					

NAME OF SCHOOL/CENTRE

CANDIDATE'S FULL NAME (FIRST, MIDDLE, LAST)

"*"Barcode Area"*"
Current Bar Code

DATE OF BIRTH | D | D | M | M | Y | Y | Y | Y |

SIGNATURE _____

"*"Barcode Area"*"
Sequential Bar Code

DO NOT WRITE ON THIS PAGE

FORM TP 2019306

TEST CODE **02134032**

MAY/JUNE 2019

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 032

ALGEBRA, GEOMETRY AND CALCULUS

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of ONE question.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

Module 1

Answer this question.

1. Let **p** and **q** be two propositions.

(a) (i) State the converse of $(\mathbf{p} \vee \mathbf{q}) \longrightarrow (\mathbf{q} \vee \sim \mathbf{p})$.

[1 mark]

(ii) Show that the contrapositive of the inverse of $(\mathbf{p} \wedge \mathbf{q}) \longrightarrow (\mathbf{q} \vee \sim \mathbf{p})$ is the converse of $(\mathbf{p} \wedge \mathbf{q}) \longrightarrow (\mathbf{q} \vee \sim \mathbf{p})$.

[3 marks]

GO ON TO THE NEXT PAGE

(b) Solve the equation $\log_4 (x^2 + 1) - \log_2 (2x - 1) = 0$.

[8 marks]

GO ON TO THE NEXT PAGE

(c) An operation $*$ is defined as $x * y = \dfrac{xy}{x + y - k}$ where $x + y \neq k$ and $x, y, k \in \mathbf{R}$. Show that $*$ is associative.

[8 marks]

Total 20 marks

GO ON TO THE NEXT PAGE

SECTION B

Module 2

Answer this question.

2. $P(-1, 3, -2)$, $Q(1, 1, 2)$ and $R(-4, 3, 2)$ are the vertices of a triangle.

(a) (i) Determine the displacement vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

[4 marks]

(ii) Hence, determine $|\overrightarrow{PQ}|$ and $|\overrightarrow{PR}|$.

[4 marks]

(iii) Determine the cosine of the acute angle between $|\overrightarrow{PQ}|$ and $|\overrightarrow{PR}|$.

[3 marks]

(iv) Calculate the area of the triangle PQR.

[4 marks]

GO ON TO THE NEXT PAGE

(b) Given that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$, show, **without the use of a calculator**, that the EXACT value of $\tan \frac{\pi}{12}$ is $2 - \sqrt{3}$.

[5 marks]

Total 20 marks

SECTION C

Module 3

Answer this question.

3. (a) (i) By expressing $(x - 9)$ as $(\sqrt{x} - 3)(\sqrt{x} + 3)$, determine $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$.

[3 marks]

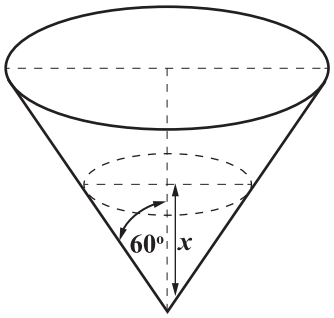
(ii) Hence, or otherwise, determine $\lim_{x \to 9} \frac{\sqrt{x}-3}{x^2-10x+9}$.

[4 marks]

(b) Determine the gradient of the curve $y = 2x^3$ at the point P on the curve at which $y = 16$.

[3 marks]

(c) The diagram below, **not drawn to scale**, represents an empty vessel in the shape of a right circular cone of semi-vertical angle 60°. Water is poured into the vessel at the rate of 10 cubic inches per second. At time t seconds after the start of the pouring of water, the height of the water in the vessel is x inches and its volume is V cubic inches.



(i) Express V in terms of t only.

[1 mark]

(ii) Express V in terms of x only.

[3 marks]

(iii) Determine, correct to 2 decimal places, the rate at which the water level is rising after 5 seconds.

[6 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.

DO NOT WRITE ON THIS PAGE

CANDIDATE'S RECEIPT

INSTRUCTIONS TO CANDIDATE:

1. **Fill in all the information requested clearly in capital letters.**

TEST CODE: | 0 | 2 | 1 | 3 | 4 | 0 | 3 | 2 |

SUBJECT: PURE MATHEMATICS – UNIT 1 – Paper 032

PROFICIENCY: ADVANCED

REGISTRATION NUMBER: | | | | | | | | | | |

FULL NAME: _____

(BLOCK LETTERS)

Signature: _____

Date: _____

2. **Ensure that this slip is detached by the Supervisor or Invigilator and given to you when you hand in this booklet.**

3. **Keep it in a safe place until you have received your results.**

INSTRUCTION TO SUPERVISOR/INVIGILATOR:

Sign the declaration below, detach this slip and hand it to the candidate as his/her receipt for this booklet collected by you.

I hereby acknowledge receipt of the candidate's booklet for the examination stated above.

Signature: _____
Supervisor/Invigilator

Date: _____