# CARIBBEAN EXAMINATIONS COUNCIL

### CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

12 JUNE 2018 (p.m.)

**FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.**

TEST CODE  | 0 | 2 | 1 | 3 | 4 | 0 | 3 | 2 |

SUBJECT    PURE MATHEMATICS – UNIT 1 – Paper 032

PROFICIENCY    ADVANCED

REGISTRATION NUMBER

SCHOOL/CENTRE NUMBER

NAME OF SCHOOL/CENTRE

CANDIDATE'S FULL NAME (FIRST, MIDDLE, LAST)

DATE OF BIRTH | D | D | M | M | Y | Y | Y | Y |

SIGNATURE _____

DO NOT WRITE ON THIS PAGE

**C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L**

**CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®**

**PURE MATHEMATICS**

**UNIT 1 – Paper 032**

**ALGEBRA, GEOMETRY AND CALCULUS**

*1 hour 30 minutes*

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### READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.

2. Each section consists of ONE question.

3. Answer ALL questions.

4. Write your answers in the spaces provided in this booklet.

5. Do NOT write in the margins.

6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.

7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**

8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

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**Examination Materials Permitted**

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

**DO  NOT  TURN  THIS  PAGE  UNTIL  YOU  ARE  TOLD  TO  DO  SO.**

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**SECTION A**

**Module 1**

**Answer this question.**

**1.** (a) Two propositions, **r** and **s** are given as

**r**: It is Monday
**s**: It is raining.

Express the following statements in symbolic form:

(i) It is Monday and it is raining.

(ii) It is not Monday or it is not raining.

**[2 marks]**

(b) (i) Given that **p** and **q** are two propositions, complete the truth table below.

| p | q | ~ p | ~ q | p ∨ q | ~ (p) ∧ ~ (q) | (p ∨ q) ∧ [ ~ (p) ∧ ~ (q) ] |
|---|---|-----|-----|-------|---------------|------------------------------|
| T | T | | | | | |
| T | F | | | | | |
| F | T | | | | | |
| F | F | | | | | |

**[4 marks]**

(ii) Hence, state whether (**p** ∨ **q**) ∧ [ ~ (**p**) ∧ ~ (**q**)] is a tautology or a contradiction. **Justify your response**.

**[2 marks]**

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(c)     Prove by mathematical induction that $2^{n+1} + 3^{2n-1}$ is a multiple of 7 for all positive integers $n \geq 1$ .

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**[7 marks]**

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(d)    Solve the equation $\log_x (2x - 3) + \log_x (4) = 2$ .

**[5 marks]**

**Total 20 marks**

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- 8 -

## SECTION B

### Module 2

**Answer this question.**

**2.** (a) A line, $L_1$, has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and

a line, $L_2$, has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ where $\alpha$ and $\beta$ are scalar parameters.

Given that $L_1$ and $L_2$ meet at a point A, determine

  (i) the coordinates of A

**[3 marks]**

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(ii) the angle between line $L_1$ and $L_2$, correct to 3 significant figures.

**[3 marks]**

(iii) Show that the vector $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is perpendicular to both $L_1$ and $L_2$.

**[3 marks]**

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(iv)    Hence, determine the vector equation of the plane through the point $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$, which

is perpendicular to the vector $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

**[3 marks]**

GO ON TO THE NEXT PAGE

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(b)     Solve the equation $\sin x + \cos 2x = 1$, where $0 \leq x \leq 2\pi$.

**[6 marks]**

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(c)     Determine the Cartesian equation of the curve with parametric equations

$x = 1 + t^2, y = 1 + t$ .

**[2 marks]**
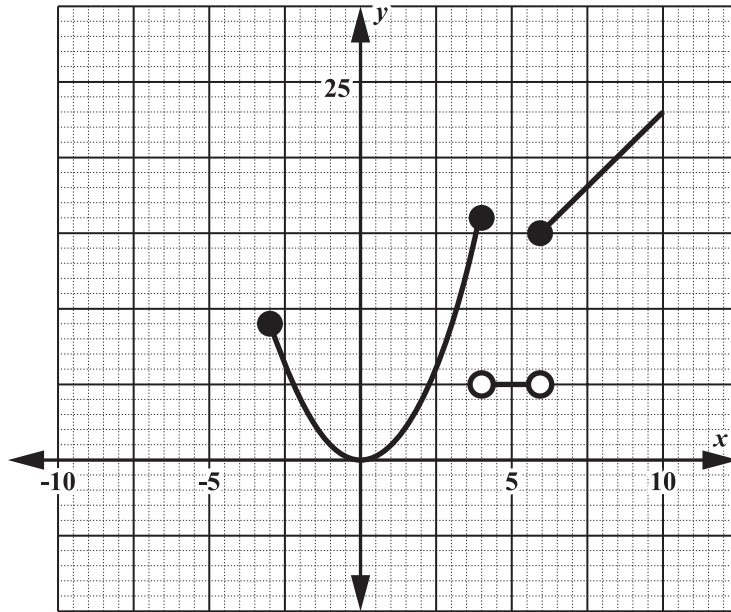
**Total 20 marks**

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## SECTION C

### Module 3

**Answer this question.**

**3.** (a) The diagram below shows the graph of $y = f(x)$.



(i) Use the graph to find $f(6)$.

**[1 mark]**

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(ii)    Determine whether the function is continuous at $x = 6$.  **Justify your response**.

**[2 marks]**

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(b)    Using **first principles**, differentiate $f(x) = \sin 2x$.

**[6 marks]**

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(c)     The diagram below (not drawn to scale) shows the curve $y = x^2$ and the line $y = x + 6$.
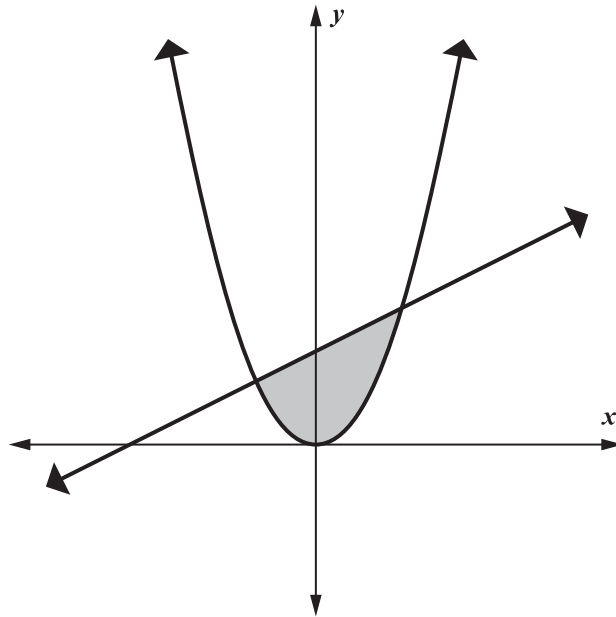


(i)     Determine the points of intersection of the curve $y = x^2$ and the line $y = x + 6$.

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**[7 marks]**

(ii) Determine the area of the region bounded between the curve $y = x^2$ and the line $y = x + 6$.

**[4 marks]**

**Total 20 marks**

**END OF TEST**

**IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.**

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**EXTRA SPACE**

**If you use this extra page, you MUST write the question number clearly in the box provided.**

**Question No.** ☐

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DO NOT WRITE ON THIS PAGE

# CANDIDATE'S  RECEIPT

## INSTRUCTIONS TO CANDIDATE:

**1. Fill in all the information requested clearly in capital letters.**

TEST CODE: | 0 | 2 | 1 | 3 | 4 | 0 | 3 | 2 |

SUBJECT:        PURE MATHEMATICS – UNIT 1 – Paper 032

PROFICIENCY:        ADVANCED

REGISTRATION NUMBER: | | | | | | | | | | |

FULL NAME: _____
**(BLOCK LETTERS)**

Signature: _____

Date: _____

**2. Ensure that this slip is detached by the Supervisor or Invigilator and given to you when you hand in this booklet.**

**3. Keep it in a safe place until you have received your results.**

## INSTRUCTION TO SUPERVISOR/INVIGILATOR:

Sign the declaration below, detach this slip and hand it to the candidate as his/her receipt for this booklet collected by you.

I hereby acknowledge receipt of the candidate's booklet for the examination stated above.

Signature: _____
Supervisor/Invigilator

Date: _____