

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 032

ALGEBRA, GEOMETRY AND CALCULUS

*1 hour 30 minutes***READ THE FOLLOWING INSTRUCTIONS CAREFULLY.**

1. This examination paper consists of THREE sections.
2. Each section consists of ONE question.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – Revised 2012

Mathematical instruments

Silent, non-programmable, electronic calculator

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SECTION A

Module 1

Answer this question.

1. (a) (i) Using the substitution $y = e^x$, or otherwise, solve the equation $e^{2x} - 21e^x - 100 = 0$.

[6 marks]

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- (ii) Solve the inequality $|x + 5| \geq |3x + 2|$.

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[4 marks]

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- (b) Let \mathbf{p} and \mathbf{q} be two propositions. Prove that $(\mathbf{p} \vee \mathbf{q}) \wedge \sim \mathbf{p} \equiv \sim \mathbf{p} \wedge \mathbf{q}$.

[6 marks]

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- (c) Use a direct proof to show that the sum of two odd numbers is even.

[4 marks]

Total 20 marks

SECTION B

Module 2

Answer this question.

2. (a) (i) Express $\cos \theta - \sqrt{3} \sin \theta$ in the form $r \cos (\theta + \alpha)$, where $r > 0$ and $0 < \alpha < \frac{\pi}{2}$.

[4 marks]

- (ii) Hence, or otherwise, find the general solution of the equation

$$\cos \theta - \sqrt{3} \sin \theta = 0.$$

[4 marks]

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- (b) Two lines L_1 and L_2 are defined parametrically by

$$L_1: x = 1 + t, y = 2 + 4t, z = -t$$

and

$$L_2: x = 2u, y = 3 + u, z = -2 + 3u$$

where t and u are scalars.

- (i) Find the vector equations of L_1 and L_2 .

[3 marks]

- (ii) Hence, or otherwise, determine whether the lines intersect, are parallel or are skewed.

Show ALL working to support your answer.

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[9 marks]

Total 20 marks

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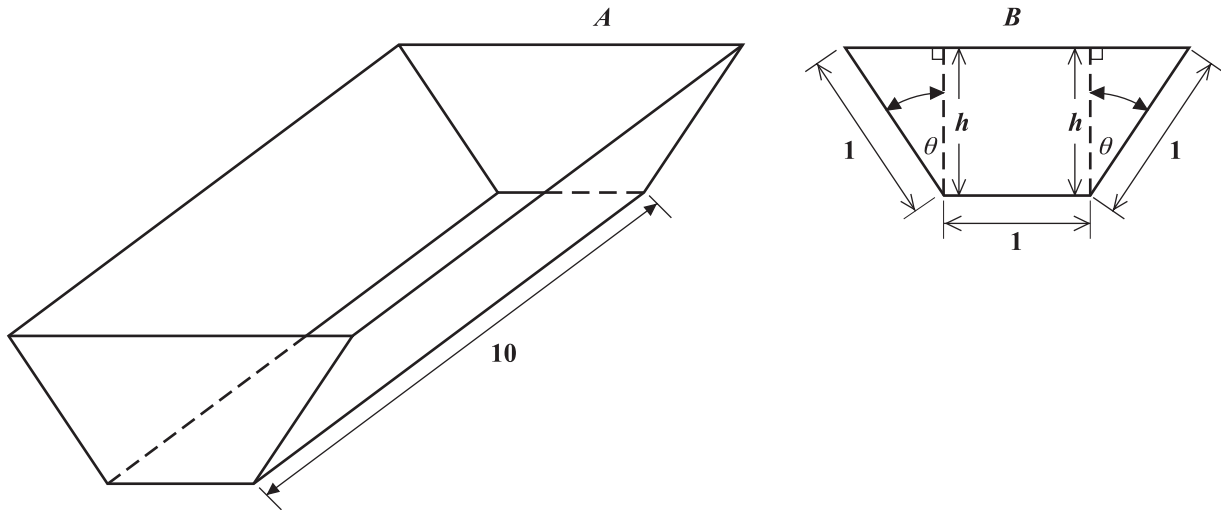
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SECTION C

Module 3

Answer this question.

3. (a) Diagram A below, **not drawn to scale**, shows the design of a trough. The cross-section of the trough, which has the shape of a trapezium, is shown in Diagram B. All lengths are in metres.



The trough must be made using the dimensions shown, but the angle θ may vary.

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- (i) Given that the volume, V , is the product of the cross-sectional area and the length of the trough, show that

$$V = 10 \cos\theta (1 + \sin\theta)$$

[5 marks]

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- (ii) Hence, or otherwise, determine the MAXIMUM possible volume of the trough.

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[10 marks]

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- (b) Calculate the area enclosed between the curve $y = x^2$ and the line $y = 4$.

[5 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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