TEST CODE **02134020**

FORM TP 2015265

MAY/JUNE 2015

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

2 hours 30 minutes

12 MAY 2015 (p.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Answer ALL questions from the THREE sections.
3. Each section consists of TWO questions.
4. Write your solutions, with full working, in the answer booklet provided.
5. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

Module 1

Answer BOTH questions.

1. (a) Let **p** and **q** be any two propositions.

(i) State the inverse and the contrapositive of the statement **p → q**. **[2 marks]**

(ii) Copy and complete the table below to show the truth table for

p → q and **~q → ~p**.

p	q	~p	~q	p → q	~q → ~p
T	*T*				
T	*F*				
F	*T*				
F	*F*				

[4 marks]

(iii) Hence, state whether the compound statements **p → q** and **~q → ~p** are logically equivalent. **Justify your response.** **[2 marks]**

(b) The polynomial $f(x) = x^3 + px^2 - x + q$ has a factor $(x - 5)$ and a remainder of 24 when divided by $(x - 1)$.

(i) Find the values of p and q. **[4 marks]**

(ii) Hence, factorize $f(x) = x^3 + px^2 - x + q$ completely. **[5 marks]**

(c) Given that $S(n) = 5 + 5^2 + 5^3 + 5^4 + ... + 5^n$, use mathematical induction to prove that

$$4S(n) = 5^{n+1} - 5 \text{ for } n \in N.$$ **[8 marks]**

Total 25 marks

2. (a) The relations $f : A \rightarrow B$ and $g : B \rightarrow C$ are functions which are both one-to-one and onto.

Show that $(g \circ f)$ is

(i) one-to-one **[4 marks]**

(ii) onto. **[4 marks]**

(b) Solve EACH of the following equations:

(i) $3 - \frac{4}{(9)^x} - \frac{4}{(81)^x} = 0$ **[7 marks]**

(ii) $| 5x - 6 | = x + 5$ **[5 marks]**

(c) The population growth of bacteria present in a river after time, t hours, is given by

$$N = 300 + 5^t.$$

Determine

(i) the number of bacteria present at $t = 0$ **[1 mark]**

(ii) the time required to triple the number of bacteria. **[4 marks]**

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) (i) Show that $\cos 3x = 4\cos^3 x - 3\cos x$. **[6 marks]**

(ii) Hence, or otherwise, solve

$$\cos 6x - \cos 2x = 0 \text{ for } 0 \leq x \leq 2\pi.$$ **[9 marks]**

(b) (i) Express $f(2\theta) = 3\sin 2\theta + 4\cos 2\theta$ in the form $r\sin(2\theta + \alpha)$ where

$$r > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}.$$ **[6 marks]**

(ii) Hence, or otherwise, find the maximum and minimum values of $\dfrac{1}{7 - f(\theta)}$.

[4 marks]

Total 25 marks

4. (a) The circles C_1 and C_2 are defined by the parametric equations as follows:

C_1: $x = \sqrt{10}\cos\theta - 3$; $y = \sqrt{10}\sin\theta + 2$

C_2: $x = 4\cos\theta + 3$; $y = 4\sin\theta + 2$.

(i) Determine the Cartesian equations of C_1 and C_2 in the form $(x - a)^2 + (y - b)^2 = r^2$. **[4 marks]**

(ii) Hence or otherwise, find the points of intersection of C_1 and C_2. **[9 marks]**

(b) A point $P\ (x, y)$ moves so that its distance from the fixed point (0, 3) is two times the distance from the fixed point (5, 2). Show that the equation of the locus of the point $P\ (x, y)$ is a circle. **[12 marks]**

Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5. (a) Let f be a function defined as

$$f(x) = \begin{cases} \dfrac{\sin(ax)}{x} & \text{if } x \neq 0, \quad a \neq 0 \\ 4 & \text{if } x = 0 \end{cases}$$

If f is continuous at $x = 0$, determine the value of a. **[4 marks]**

(b) **Using first principles,** determine the derivative of $f(x) = \sin(2x)$. **[6 marks]**

(c) If $y = \dfrac{2x}{\sqrt{1 + x^2}}$ show that

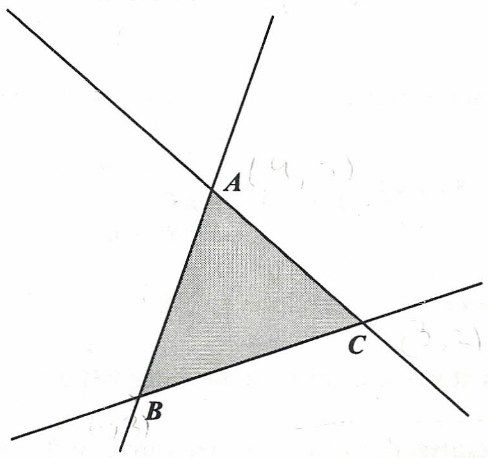
(i) $x \dfrac{dy}{dx} = \dfrac{y}{1 + x^2}$ **[7 marks]**

(ii) $\dfrac{d^2y}{dx^2} + \dfrac{3y}{(1 + x^2)^2} = 0.$ **[8 marks]**

Total 25 marks

6. (a) The diagram below **(not drawn to scale)** shows the region bounded by the lines

$$y = 3x - 7, \quad y + x = 9 \quad \text{and} \quad 3y = x + 3.$$



(a) (i) Show that the coordinates of A, B and C are (4, 5), (3, 2), and (6, 3) respectively. **[5 marks]**

(ii) Hence, use integration to determine the area bounded by the lines. **[6 marks]**

(b) The gradient function of a curve $y = f(x)$ which passes through the point (0, –6) is given by $3x^2 + 8x - 3$.

(i) Determine the equation of the curve. **[3 marks]**

(ii) Find the coordinates and nature of the stationary point of the curve in (b) (i) above. **[8 marks]**

(iii) Sketch the curve in (b) (i) by clearly labelling the stationary points. **[3 marks]**

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.