FORM TP 2022328

TEST CODE **02134032**

MAY/JUNE 2022

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 032

ALGEBRA, GEOMETRY AND CALCULUS

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of ONE question.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

"*"Barcode Area"*"
Sequential Bar Code

SECTION A

Module 1

Answer this question.

1. (a) (i) Let **p**, **q** and **r** be any three propositions. Complete the truth table below.

p	**q**	**r**	**q ∧ r**	**p ∨ q**	**p ∨ r**	**p ∨ (q ∧ r)**	**(p ∨ q) ∧ (p ∨ r)**
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

[4 marks]

(ii) Hence, state whether the statements **p ∨ (q ∧ r)** and **(p ∨ q) ∧ (p ∨ r)** are logically equivalent. **Justify your response**.

.....

.....

.....

.....

[2 marks]

GO ON TO THE NEXT PAGE

(b) Using mathematical induction, prove that $1 + 3 + 5 + 7 + ... + (2n - 1) = n^2$ for all natural numbers n.

[7 marks]

(c) Solve the equation $\log_{3x}(3x + 2) + \log_{3x} 2 = 2$, leaving your answer in surd form.

[7 marks]

Total 20 marks

SECTION B

Module 2

Answer this question.

2. (a) Solve the equation $2 \sin^2 x = 3 \cos x + 3$, $0 \leq x < 2\pi$.

[6 marks]

(b) The points P, Q and R are (1, 3, 2), (2, 4, –1) and (3, –2, 1), respectively.

(i) Determine the angle QPR.

[7 marks]

(ii) Hence, state the relationship between the lines PQ and PR.

.....

.....

[1 mark]

(c) Determine the equation of the plane through the point (2, –5, 6) which is perpendicular to the vector $\begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix}$.

[6 marks]

Total 20 marks

SECTION C

Module 3

Answer this question.

3. (a) **Using first principles**, show that the derivative of $g(x) = \frac{1}{x}$ is $-\frac{1}{x^2}$.

[6 marks]

(b) A sheet of metal has dimensions 24 cm by 9 cm. An open rectangular box is made from the metal by cutting out squares of side x, from each corner.

(i) Write an expression, in terms of x, for the volume of the box.

[2 marks]

(ii) Determine the MAXIMUM volume of the box.

[6 marks]

(c) The equations of a curve are given parametrically as $x = 2t - 1$, $y = 4t^2 - 2t$.

(i) Write the equation of the curve using rectangular coordinates, that is, in the form $y = f(x)$.

[3 marks]

(ii) Hence, determine the equation of the tangent to the curve at the point (1, 2).

[3 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.