CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

"*"Barcode Area"*"
Front Page Bar Code

08 JUNE 2021 (p.m.)

FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

TEST CODE | 0 | 2 | 1 | 3 | 4 | 0 | 3 | 2 |

SUBJECT PURE MATHEMATICS – UNIT 1 – Paper 032

PROFICIENCY ADVANCED

REGISTRATION NUMBER | | | | | | | | | | |

SCHOOL/CENTRE NUMBER					

NAME OF SCHOOL/CENTRE

CANDIDATE'S FULL NAME (FIRST, MIDDLE, LAST)

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Current Bar Code

DATE OF BIRTH | D | D | M | M | Y | Y | Y | Y |

SIGNATURE _____

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FORM TP 2021328

TEST CODE **02134032**

MAY/JUNE 2021

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 032

ALGEBRA, GEOMETRY AND CALCULUS

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of ONE question.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable electronic calculator

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SECTION A

Module 1

Answer this question.

1. (a) The roots of the cubic equation $2x^3 - 3x^2 - x + 8 = 0$ are α, β and γ. Determine the equation which has roots α^2, β^2 and γ^2.

Note: $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$
$(\alpha\beta)^2 + (\beta\gamma)^2 + (\alpha\gamma)^2 = (\alpha\beta + \beta\gamma + \alpha\gamma)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)$

[7 marks]

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(b) Prove by mathematical induction that $1 + 3 + 3^2 + 3^3 + ... + 3^{n-1} = \frac{3^n - 1}{2}$ for all positive integers $n \geq 1$.

[8 marks]

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(c) Solve the equation $\log_3 (5x - 2) + \log_9 (5x - 2) = 3$, where $x > \frac{2}{5}$.

[5 marks]

Total 20 marks

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SECTION B

Module 2

Answer this question.

2. (a) $P(1, 5, 2)$, $Q(2, 6, 3)$ and $R(4, 2, -1)$ are the vertices of a triangle.

(i) Determine the displacement vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

[4 marks]

(ii) Hence, determine $|\overrightarrow{PQ}|$ and $|\overrightarrow{PR}|$.

[4 marks]

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(iii) Determine the acute angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

[4 marks]

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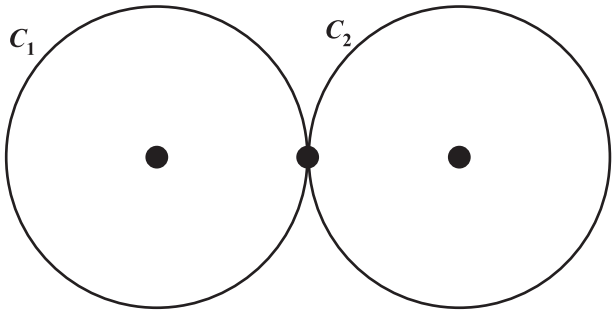
(b) The following diagram (**not drawn to scale**) shows two circles, C_1 and C_2, touching at the point $P(-4, 3)$. The equation of the circle C_1 is given by $x^2 + y^2 + 14x - 6y + 49 = 0$. The radius of C_2 is 3. Determine the equation of the circle C_2.



[8 marks]

Total 20 marks

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SECTION C

Module 3

Answer this question.

3. (a) Calculate the volume of the solid obtained by rotating the region bounded by the curves $y = x^3$, $y = 8$ and $x = 0$ around the y-axis.

[6 marks]

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(b) The voltage, V, in a circuit can be represented by the equation $\frac{dV}{dt} - \frac{V}{2} = 0$.

Given that $V = 25$ volts and $t = 0$, determine an expression for V in terms of t.

Note that $\int \frac{1}{x}\, dx = \ln x + c$.

[6 marks]

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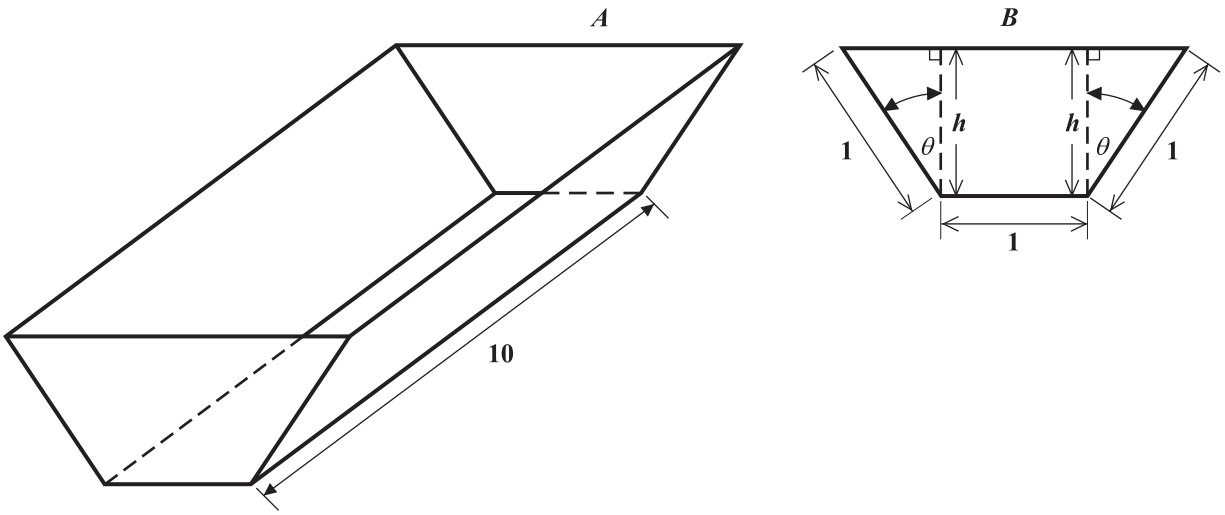
(c) Diagram *A* below (**not drawn to scale**) shows the design of a trough. The cross-section of the trough, which has the shape of a trapezium, is shown in Diagram *B*. All lengths are in metres.



The trough must be made using the dimensions shown, but the angle θ may vary.

(i) Show that the cross-sectional area, X, of the trough is $X = \cos\theta\,(1 + \sin\theta)$.

[3 marks]

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(ii) Hence, or otherwise, determine the acute angle, θ, that would **maximize** the volume of the trough.

[5 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.

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CANDIDATE'S RECEIPT

INSTRUCTIONS TO CANDIDATE:

1. **Fill in all the information requested clearly in capital letters.**

TEST CODE: | 0 | 2 | 1 | 3 | 4 | 0 | 3 | 2 |

SUBJECT: PURE MATHEMATICS – UNIT 1 – Paper 032

PROFICIENCY: ADVANCED

REGISTRATION NUMBER: | | | | | | | | | | |

FULL NAME: _____

(BLOCK LETTERS)

Signature: _____

Date: _____

2. **Ensure that this slip is detached by the Supervisor or Invigilator and given to you when you hand in this booklet.**

3. **Keep it in a safe place until you have received your results.**

INSTRUCTION TO SUPERVISOR/INVIGILATOR:

Sign the declaration below, detach this slip and hand it to the candidate as his/her receipt for this booklet collected by you.

I hereby acknowledge receipt of the candidate's booklet for the examination stated above.

Signature: _____
Supervisor/Invigilator

Date: _____