CAPE Unit 2 Test 3 (2020) Preview exercise

1 (a) arranging 8 objects with 2 T's and 2 Ss

$$\frac{8!}{2!\,2!}$$

(b)(i) $\boxed{T}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{T}$

arranging 6 object with 2 Ss

$$\frac{6!}{2!}$$

(ii) group $\boxed{E\ S\ S}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }$

arrange 6 objects with 2Ts $= \dfrac{6!}{2!}$

arrange E SS $= \dfrac{3!}{2!}$

so total number of arrangement

$$\frac{6!}{2!} \times \frac{3!}{2!}$$

(iii) $\boxed{E}\ \boxed{\ }\ \boxed{S}\ \boxed{\ }\ \boxed{S}\ \boxed{\ }\ \boxed{I}\ \boxed{\ }$

arrange 4 "blank" spaces with 2Ts $= \dfrac{4!}{2!}$

arrange E SS I $= \dfrac{4!}{2!}$

but we can have

$\boxed{\ }\ \boxed{E}\ \boxed{\ }\ \boxed{S}\ \boxed{\ }\ \boxed{S}\ \boxed{\ }\ \boxed{I}$

So total number of arrangement $= 2 \times \dfrac{4!}{2!} \times \dfrac{4!}{2!}$

2 (i) 6 men or 6 women (OR = +)

$^7C_6 + {^6C_6}$

(ii) 3 men and 3 women (and = ×)

$^7C_3 \times {^6C_3}$

(iii) Brother (A) but not (B) $= {^{11}C_5}$

OR Brother B but not A $= {^{11}C_5}$

So $^{11}C_5 + {^{11}C_5}$

3

	$h<2$	$2<h<4$	$h>4$	
Pale green	2	15	3	20
off white	11	21	8	40
	13	36	11	60

(a) Prob (off white) $= \frac{40}{60}$

(b) $Pr(h>4) = \frac{11}{60}$

(c) Pr (pale green and less 4 cm) $= \frac{15+2}{60} = \frac{17}{60}$

(d) $Pr(h>2/G) = \frac{P(h>2 \cap G)}{P(\text{Green})} = \frac{\frac{18}{60}}{\frac{20}{60}} = \frac{18}{20}$

4 (a) $2\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} x+2 & 4 \\ x+1 & 2 \end{vmatrix} + (x-1)\begin{vmatrix} x+2 & 3 \\ x+1 & 1 \end{vmatrix} = 0$

$$4 + 2x - 2x^2 + x + 1 = 0$$

$$-2x^2 + 3x + 5 = 0$$

$$2x^2 - 3x - 5 = 0$$

$$(2x-5)(x+1) = 0$$

$$x = \frac{5}{2} \qquad x = -1$$

4(b) (i) $\left[\begin{array}{ccc|c} 1 & -1 & -1 & 3 \\ 2 & 1 & -1 & 10 \\ 1 & -2 & 3 & 6 \end{array}\right]$

(ii)

$$\begin{array}{c} \\ R_2 - 2R_1 \\ R_3 - R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 3 \\ 0 & 3 & 1 & 4 \\ 0 & -1 & 4 & 3 \end{array}\right]$$

$$\begin{array}{c} \\ \\ 3R_3 + R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & 13 & 13 \end{array}\right]$$

$$13z = 13 \Rightarrow z = 1$$

$$3y + z = 4$$
$$3y + 1 = 4 \Rightarrow y = 1$$

$$x - y - z = 3 \Rightarrow x = 5$$
$$x - 1 - 1 = 3$$

5/ $$\frac{dc}{dx} + 2c = 10x$$

$$\int 2\,dx = 2x \Rightarrow \text{Integrating factor} = e^{2x}$$

$$ce^{2x} = \int 10x\,e^{2x}\,dx = 10\int x e^{2x}\,dx$$

$$= 10\left[\frac{xe^{2x}}{2} - \frac{1}{2}\int e^{2x}\,dx\right]$$

$$ce^{2x} = 10\left[\frac{xe^{2x}}{2} - \frac{1}{4}e^{2x} + A\right]$$

$$ce^{2x} = 5xe^{2x} - \frac{5}{2}e^{2x} + A$$

$$c = 5x - \frac{5}{2} + Ae^{-2x}$$

(ii) when $x = 0$ $c = 100$

$$100 = -\frac{5}{2} + A \Rightarrow A = \frac{205}{2}$$

$$c = 5x - \frac{5}{2} + \frac{205}{2}e^{-2x}$$

6/ $y'' + 2y' + 5y = 4\sin 2t$

auxilary equation : $m^2 + 2m + 5 = 0$

$$m = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm \sqrt{-16}}{2} = -1 \pm 2i$$

Complementary function: $y = e^{-t}(A\cos 2t + B\sin 2t)$

for Particular integral try

$$y = C\cos 2t + D\sin 2t$$

$$y' = -2C\sin 2t + 2D\cos 2t$$

$$y'' = -4C\cos 2t - 4D\sin 2t$$

substituting

$$-4C\cos 2t - 4D\sin 2t$$
$$4D\cos 2t - 4C\sin 2t$$
$$5C\cos 2t + 5D\sin 2t$$

$$\Rightarrow \quad C + 4D = 0$$
$$-4C + D = 4$$

$$C + 4D = 0$$
$$-16C + 4D = 16$$

$$-17C = 16 \Rightarrow \quad C = -\frac{16}{17}$$

and $\quad B = \frac{4}{17}$

(iii) So general solution

$$y = e^{-t}\left[A\cos 2t + B\sin 2t\right] - \frac{16}{17}\cos 2t + \frac{4}{17}\sin 2t$$

when $t=0$ $\quad y = \frac{4}{100}$

$t=0 \quad y' = 0$

$$\frac{4}{100} = A - \frac{16}{17} \implies A = \frac{4}{100} + \frac{16}{17} = \frac{417}{425}$$

$$y' = e^{-t}\left[-2A\sin 2t + 2B\cos 2t\right]$$
$$- e^{-t}\left[A\cos 2t + B\sin 2t\right]$$
$$+ \frac{32}{17}\sin 2t + \frac{8}{17}\cos 2t$$

$$0 = 2B - A + \frac{8}{17}$$

$$B = \frac{\frac{417}{425} - \frac{8}{17}}{2} = \frac{217}{850}$$

$$y(t) = e^{-t}\left(\frac{417}{425}\cos 2t + \frac{217}{850}\sin 2t\right) - \frac{16}{17}\cos 2t + \frac{4}{17}\sin 2t$$