HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2018
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 2 – TEST 3 PREVIEW
1 hour 20 minutes

This examination paper consists of 2 pages.
This paper consists of 5 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write in ink.
2. Write your name clearly on each sheet of paper used.
3. Answer **ALL** questions.
4. Do **NOT** do questions beside one another.
5. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

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1. (a) A committee of 4 persons is to be selected from a group of 6 males and 6 females. Determine the number of ways the committee may be formed if it is to have at least one male. [480] [3]

 (b) (i) Find the number of arrangements of all letters of the word COMBINATION. [2]
 [4 989 600]

 (ii) Find the probability that the two letters, N, are next to each other. $\left[\frac{2}{11}\right]$ [3]

 (c) Find how many even numbers between 3000 and 7000 can be written down using the digits 1, 3, 6, 8 if no digit can occur more than once in any number. [6] [3]

Total 11 marks

2. (a) The probability that it rains on any day in Barbados during March is 0.3. The probability that Dennis will get to school late is 0.6 when it rains and 0.5 when it does not rain. Given that Dennis got to school late on a particular day in March, find the probability that it rained on that day. [0.339] [6]

 (b) Events A and B are such that $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{3}$ and $P(A|B) = \frac{1}{5}$.

 (i) Find $P(A \cap B)$. $\left[\frac{1}{15}\right]$ [2]

 (ii) Find $P(A \cup B)$. $\left(\frac{13}{15}\right)$ [2]

 (iii) State with a reason whether A and B are independent events.[not independent][2]

PLEASE TURN OVER

Total 12 marks

3. The matrix $\boldsymbol{D}$ is given by $= \begin{pmatrix} 1 & 4 & 2 \\ 3 & k & 3 \\ 2 & k & 1 \end{pmatrix}$.

(i) Show that $\boldsymbol{D}$ is non-singular for all values of k. [3]

(ii) Find $\boldsymbol{D}^{-1}$ in terms of k. [5]

(ii) Hence, or otherwise, solve the equations

$$\begin{aligned} x + 4y + 2z &= 25 \\ 3x + ky + 3z &= 3 \\ 2x + ky + z &= 2 \end{aligned}$$

giving your solution in terms of k. [4]

Total 12 marks

4. A system of equations is given by

$$\begin{aligned} 2x - y + 3z &= 17 \\ x + 2y - z &= -4 \\ 3x + y + 2z &= k \end{aligned}$$

where k is a real number.

(i) Write the system in matrix form. [1]

(ii) Write down the augmented matrix. [1]

(iii) Reduce the augmented matrix to echelon form. [3]

(iv) Deduce the value of k which the system is consistent. [1]

(v) Find ALL solutions corresponding to this value of k. [3]

Total 9 marks

5. (a) Solve the differential equation

$$\frac{dy}{dx} + 3y = 5$$

given that $y = 2$ when $x = 0$. [7]

(b) The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 4x$$

(i) Find the complementary function. [3]

(ii) Hence find the general solution of the differential equation. [6]

Total 16 marks

End of Examination

Answers

3 (i) determinant of D =12

(ii) $D^{-1} = \frac{1}{12}\begin{pmatrix} -2k & 2k-4 & 12-2k \\ 3 & -3 & 3 \\ k & 8-k & k-12 \end{pmatrix}$

(iii) $x = 1 - 4k \quad y = 6 \quad z = 2k$

4. (i) $\begin{pmatrix} 2 & -1 & 3 \\ 1 & 2 & -1 \\ 3 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 17 \\ -4 \\ k \end{pmatrix}$

(ii) $\begin{pmatrix} 2 & -1 & 3 & 17 \\ 1 & 2 & -1 & -4 \\ 3 & 1 & 2 & k \end{pmatrix}$

(iii) $\begin{pmatrix} 2 & -1 & 3 & 17 \\ 0 & -5 & -5 & -25 \\ 0 & 0 & 0 & k-13 \end{pmatrix}$

(iv) $k = 13$

(v) $x = 6 - t \quad y = t - 5 \quad z = t$

5(a) $y = \frac{5}{3} + \frac{1}{3}e^{-3x}$

(b) (i) $y = Ae^{4x} + Be^{2x}$

(ii) $y = Ae^{4x} + Be^{2x} + \frac{1}{2}x + \frac{3}{8}$

END OF EXAMINATION