FORM TP 2015269

TEST CODE 02234032

MAY/JUNE 2015

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2 – Paper 032

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 hour 30 minutes

03 JUNE 2015 (a.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Answer ALL questions from the THREE sections.
3. Each section consists of ONE question.
4. Write your solutions, with full working, in the answer booklet provided.
5. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

Module 1

Answer this question.

1. (a) A complex number z_1 is such that $|z_1| = 2$ and $\arg z_1 = \frac{3\pi}{4}$.

(i) Identify the coordinates of z_1 on an Argand diagram. **[3 marks]**

(ii) On the same axes, connect z_1 to the origin with a line segment and label the angle that represents $\arg z_1$. **[2 marks]**

(iii) On the same axes, sketch the locus of the point z_2 which moves in the complex plane such that $|z_1 - z_2| = 1$. **[2 marks]**

(b) Use the trapezium rule with five ordinates to find an approximate value of

$$\int_0^2 \sqrt{4 + x^3}\, dx.$$ **[6 marks]**

(c) (i) Determine $\int \frac{\sin^{-1}(\frac{x}{2})}{\sqrt{4 - x^2}}\, dx$. **[5 marks]**

(ii) Hence, calculate $\int_0^1 \frac{\sin^{-1}(\frac{x}{2})}{\sqrt{4 - x^2}}\, dx$. **[2 marks]**

Total 20 marks

SECTION B

Module 2

Answer this question.

2. (a) (i) Determine the Taylor series expansion of

$$f(x) = e^x \cos x$$

centred at $\frac{\pi}{2}$ up to and including the first two non-zero terms. **[4 marks]**

(ii) Hence, estimate $f(\frac{\pi}{6})$. **[2 marks]**

(b) The twentieth term of an arithmetic progression is 35 and the sum of the first 19 terms is 285. Calculate the sum of the first five terms. **[7 marks]**

(c) The numbers $n - 4$, $n + 2$, $3n + 1$ are consecutive terms of a geometric sequence. Given that the corresponding series converges, determine the common ratio. **[7 marks]**

Total 20 marks

SECTION C

Module 3

Answer this question.

3. (a) A board game involves tossing TWO fair dice and ONE fair coin. The face shown on the coin determines the action of the next player.

If a HEAD is observed on the coin, the total on the dice is as observed. If a TAIL is observed on the coin, the number on each die must be 3 or less. If any of the numbers is more than 3, the die is thrown again until a 1, 2 or 3 is shown.

(i) Copy and complete the table below to show the possible totals of the throws.

			DIE 1					
			1	**2**	**3**	**4**	**5**	**6**
HEAD	**DIE 2**	**1**	2	3	4	5		
		2	3	4	5	6		
		3	4	5	6	7		
		4	5	6	7	8		
		5						
		6						
TAIL		**1**						
		2						
		3						

[2 marks]

(ii) What is the probability that the sum of the numbers on the dice is EVEN on any turn in the game? **[2 marks]**

(iii) Determine the probability of obtaining a HEAD and an EVEN total on the dice. **[4 marks]**

(iv) State, giving a reason for your answer, whether the events of obtaining a HEAD and an EVEN total on the dice are independent. **[2 marks]**

GO ON TO THE NEXT PAGE

(b) A differential equation is given as $y' + y = 2 \sin x$.

(i) Determine the general solution of the differential equation. **[8 marks]**

(ii) Hence, or otherwise, obtain the particular solution given that when $x = 0$, $y = 1$. **[2 marks]**

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.