TEST CODE **02234032**

FORM TP 2014244

MAY/JUNE 2014

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2 – Paper 032

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 hour 30 minutes

04 JUNE 2014 (a.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Answer ALL questions from the THREE sections.
3. Each section consists of ONE question.
4. Write your solutions, with full working, in the answer booklet provided.
5. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

Module 1

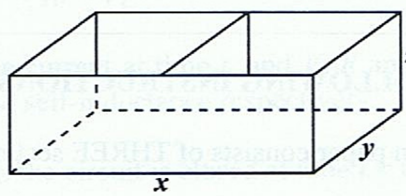
Answer this question.

1. (a) Let $I = \int_{-1}^{1} \frac{1}{1+e^{-x}}\, dx$.

(i) Use the trapezium rule with two trapezia of equal width to obtain an estimate of I. **[3 marks]**

(ii) Evaluate the integral I by means of the substitution $u = e^x$. **[7 marks]**

(b) The diagram below **(not drawn to scale)** shows an open rectangular box with a partition in the middle.



The dimensions of the box, measured in centimetres, are x, y, and z. The volume of the box is 384 cm^3.

(i) The pieces from which the box is assembled are cut from a flat plank of wood. Show that the TOTAL area of the pieces cut from the plank, A cm^2, is given by

$$A = xy + \frac{768}{y} + \frac{1152}{x}.$$ **[5 marks]**

(ii) The minimum value of A occurs where $\frac{\partial A}{\partial x} = 0$ and $\frac{\partial A}{\partial y} = 0$ simultaneously.

a) Determine $\frac{\partial A}{\partial x}$ and $\frac{\partial A}{\partial y}$. **[3 marks]**

b) Hence, show that the equations $\frac{\partial A}{\partial x} = 0$ and $\frac{\partial A}{\partial y} = 0$ are both satisfied by $x = 12, y = 8$. **[2 marks]**

Total 20 marks

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SECTION B

Module 2

Answer this question.

2. (a) (i) Show that $\frac{1}{2r-1} - \frac{1}{2r+1} = \frac{2}{4r^2-1}$. **[3 marks]**

(ii) Hence, or otherwise, show that $\sum_{r=1}^{n} \frac{2}{4r^2-1} = \frac{2n}{2n+1}$. **[5 marks]**

(b) An arithmetic progression is such that the fifth and tenth partial sums are $S_5 = 60$ and $S_{10} = 202$ respectively.

(i) Calculate the first term, a, and the common difference, d. **[5 marks]**

(ii) Hence, or otherwise, calculate the 15th term, u_{15}. **[2 marks]**

(c) (i) Show that the function $f(x) = e^{-x} - 2x + 3$ has a root, α, in the closed interval [1, 2]. **[2 marks]**

(ii) Apply linear interpolation ONCE in the interval [1, 2] to find an approximation to the root, α. **[3 marks]**

Total 20 marks

SECTION C

Module 3

Answer this question.

3. (a) A bag contains 2 red balls, 3 blue balls and 1 white ball. In an experiment, 2 balls are drawn at random from the bag without replacement.

(i) Use a tree diagram to show the possible events and their corresponding probabilities. **[5 marks]**

(ii) Calculate the probability that the second ball drawn is blue. **[4 marks]**

(b) The current flow in a particular circuit is defined by the differential equation

$$L\frac{di}{dt} + Ri = V,$$

where i is the current at time t, and V, R and L are constants representing the voltage, resistance and self-inductance respectively.

The switch in the circuit is closed at time $t = 0$ and $i(0) = 0$.

(i) By solving the differential equation using an appropriate integrating factor, verify that $i = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$. **[8 marks]**

(ii) The steady-state current in the circuit is $\lim_{t \to \infty} i$. Use the result of (b) (i) above to evaluate $\lim_{t \to \infty} i$. **[3 marks]**

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.