# $\mathbb{C A R I B B E A N}$ <br> EXAMINATIONS <br> COUNCIL <br> CARIBBEAN ADVANCED PROFICIENCY EXAMINATION ${ }^{\circledR}$ PURE MATHEMATICS 

UNIT 1 - Paper 032
ALGEBRA, GEOMETRY AND CALCULUS
1 hour 30 minutes

11 JUNE 2014 (p.m.)

## READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Answer ALL questions from the THREE sections.
3. Each section consists of ONE question.
4. Write your solutions, with full working, in the answer booklet provided.
5. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.

## Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) - Revised 2012
Mathematical instruments
Silent, non-programmable, electronic calculator

## SECTION A

## Module 1

## Answer this question.

1. (a) The binary operation * is defined on a set $\{e, a, b, c, d, f\}$ as shown in the table below. For example, $a * b=d$.

| $*$ | $\boldsymbol{e}$ | $\boldsymbol{a}$ | $\boldsymbol{b}$ | $\boldsymbol{c}$ | $\boldsymbol{d}$ | $\boldsymbol{f}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{e}$ | $e$ | $a$ | $b$ | $c$ | $d$ | $f$ |
| $\boldsymbol{a}$ | $a$ | $e$ | $d$ | $f$ | $b$ | $c$ |
| $\boldsymbol{b}$ | $b$ | $f$ | $e$ | $d$ | $c$ | $a$ |
| $\boldsymbol{c}$ | $c$ | $d$ | $f$ | $e$ | $a$ | $b$ |
| $\boldsymbol{d}$ | $d$ | $c$ | $a$ | $b$ | $f$ | $e$ |
| $\boldsymbol{f}$ | $f$ | $b$ | $c$ | $a$ | $e$ | $d$ |

(i) State, giving a reason, if * is commutative.
(ii) Name the identity element for the operation *.
(iii) Determine the inverse of
a) $d$
b) $c$.
(b) Let $\alpha, \beta$ and $\gamma$ be the roots of the equation $2 x^{3}+4 x^{2}+3 x-1=0$.
(i) Calculate EACH of the following:
a) $\frac{1}{\alpha^{2}}+\frac{1}{\beta^{2}}+\frac{1}{\gamma^{2}}$
[5 marks]
b) $\frac{1}{\alpha^{2} \beta^{2}}+\frac{1}{\beta^{2} \gamma^{2}}+\frac{1}{\gamma^{2} \alpha^{2}}$
(ii) Hence, or otherwise, find the equation whose roots are $\frac{1}{\alpha^{2}}, \frac{1}{\beta^{2}}, \frac{1}{\gamma^{2}}$.
(c) An answer sheet is provided for this question.

The diagram below shows the graph of the function $\mathrm{g}(x)=\frac{1}{x-1}$.

(i) On the answer sheet provided, sketch the graphs of $|\mathrm{g}(x)|$ and $\mathrm{f}(x)=x-1$, showing clearly the intercepts and the asymptotes.
(ii) Hence, or otherwise, obtain the value of $x$ such that $\mathrm{f}(x)=\mathrm{g}(x)$.

Total 20 marks

## SECTION B

## Module 2

## Answer this question.

2. (a) $\quad P(1,3,2), Q(-1,2,3)$ and $R(1,3,5)$ are the vertices of a triangle.
(i) Find the displacement vectors $\overrightarrow{P Q}$ and $\overrightarrow{P R}$.
(ii) Hence, determine
a) $\quad|\overrightarrow{P Q}|$ and $|\overrightarrow{P R}| \quad$ [3 marks]
b) the cosine of the acute angle between $\overrightarrow{P Q}$ and $\overrightarrow{P R}$ [4 marks]
c) the area of triangle $P Q R$.
(b) Given that $\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$, show without the use of a calculator, that EXACT value of $\tan \frac{\pi}{12}$ is $2-\sqrt{3}$.

## SECTION C

## Module 3

## Answer this question.

3. (a) Evaluate $\lim _{x \rightarrow 0} \frac{\sin 8 x}{2 x}$.
(b) The equation of a curve is given by

$$
y=x^{3}+x^{2}+2 .
$$

(i) Determine the coordinates of the points on the curve where the gradient is 1.
(ii) Determine the equation of the normal which intersects the curve at $(-1,2)$.
[4 marks]
(c) The diagram below (not drawn to scale) shows the design of a petal drawn on a square tile of length 1 metre.


The design may be modelled by the finite region enclosed by the curves $y=\sqrt{x}$ and $y=x^{2}$ where $x$ and $y$ are lengths measured in metres.

Calculate the area of the petal.

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Answer Sheet for Question 1 (c) Candidate Number .


