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MAY/JUNE 2013

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2 – Paper 032

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 hour 30 minutes

05 JUNE 2013 (a.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A (Module 1)

Answer this question.

1. (a) A firm measures production by the Cobb-Douglas production function

$$P(k(t), l(t)) = 20k^{\frac{1}{4}}\ l^{\frac{3}{4}}$$

where k is the capital (in millions of dollars) and l is the labour force (in thousands of workers).

Let $l = 3$ and $k = 4$.

Assume that the capital is DECREASING at a rate of \$200 000 per year and that the labour force is INCREASING at a rate of 60 workers per year.

Given that $\frac{dP}{dt} = \frac{\partial P}{\partial k} \bullet \frac{dk}{dt} + \frac{\partial P}{\partial l} \bullet \frac{dl}{dt}$, calculate $\frac{dP}{dt}$. **[6 marks]**

(b) Let $F_n(x) = \int \cos^n x\ dx$.

By rewriting $\cos^n x$ as $\cos x \cos^{n-1} x$ or otherwise, prove that

$$F_n(x) = \frac{1}{n} \cos^{n-1} x \sin x + \left(\frac{n-1}{n}\right) F_{n-2}(x).$$ **[6 marks]**

(c) Find the square root of the complex number $z = 2 + i$. **[8 marks]**

Total 20 marks

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SECTION B (Module 2)

Answer this question.

2. (a) (i) Show that the binomial expansion of $\left(1+\frac{1}{2}x\right)^4$ is

$$1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4.$$

[4 marks]

(ii) Hence, compute 1.377^4 correct to two decimal places. **[4 marks]**

(b) (i) Derive the first three non-zero terms in the Maclaurin expansion of $\ln(1+x)$. **[4 marks]**

(ii) Hence, express the Maclaurin expansion of $\ln(1+x)$ in sigma notation. **[2 marks]**

(c) A geometric series is given by

$$x + \frac{x^2}{2} + \frac{x^3}{4} + \frac{x^4}{8} + \ldots$$

(i) Determine the values of x for which the series is convergent. **[3 marks]**

(ii) Hence, or otherwise, if the series is convergent, show that $S_2 < 4$. **[3 marks]**

Total 20 marks

SECTION C (Module 3)

Answer this question.

3. (a) A system of equations $\mathbf{Ax} = \mathbf{b}$ is given by

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 3 \\ 1 & -2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -9 \\ 3 \end{pmatrix}$$

(i) Calculate $|\mathbf{A}|$. **[3 marks]**

(ii) Let the matrix $\mathbf{C} = \begin{pmatrix} 8 & 7 & -3 \\ 4 & -1 & 3 \\ 2 & -5 & -3 \end{pmatrix}$

a) Show that $\mathbf{C}^{\mathrm{T}}\mathbf{A} - 18\mathbf{I} = 0$. **[4 marks]**

b) Hence or otherwise, obtain $\mathbf{A}^{-1}$. **[2 marks]**

c) Solve the given system of equations for x, y and z. **[4 marks]**

(b) To make new words, **three** letters are selected without replacement from the word TRAVEL and are written down in the order in which they are selected.

(i) How many three-letter words may be formed? **[2 marks]**

(ii) For a three-letter word to be legal, it must have at least one vowel (that is a, e, i, o or u). What is the probability that a legal word is formed on a single attempt? **[5 marks]**

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.