TEST CODE **02134032**

FORM TP 2013234

MAY/JUNE 2013

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 032

ALGEBRA, GEOMETRY AND CALCULUS

1 hour 30 minutes

12 JUNE 2013 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 5 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A (Module 1)

Answer this question.

1. (a) Let **p** and **q** be two propositions.

(i) State the converse of $(p \wedge q) \rightarrow (q \vee \sim p)$. **[1 mark]**

(ii) Show that the contrapositive of the inverse of $(p \wedge q) \rightarrow (q \vee \sim p)$ is the converse of $(p \wedge q) \rightarrow (q \vee \sim p)$. **[3 marks]**

(b) Solve the equation $\log_2 (x + 3) = 3 - \log_2 (x + 2)$. **[5 marks]**

(c) The amount of impurity, **A**, present in a chemical depends on the time it takes to purify. It is known that $\mathbf{A} = 3e^{4t} - 7e^{2t} - 6$ at any time t minutes. Find the time at which the chemical is free of impurity (that is when $\mathbf{A} = 0$). **[6 marks]**

(d) On the same axes, sketch the graphs of $f(x) = 2x + 3$ and $g(x) = |2x + 3|$.

Show clearly ALL intercepts that may be present. **[5 marks]**

Total 20 marks

SECTION B (Module 2)

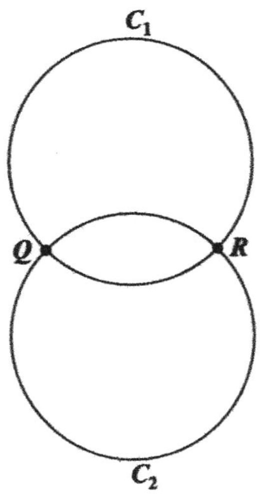
Answer this question.

2. (a) A is an acute angle and B is an obtuse angle, where $\sin(A) = \frac{4}{5}$ and $\cos(B) = -\frac{3}{5}$.

Without finding the values of angles A and B, calculate $\cos(3A)$. **[5 marks]**

(b) Solve the equation $4\cos 2\theta - 14\sin\theta = 7$ for values of θ between 0 and 2π radians. **[8 marks]**

(c) An engineer is asked to build a table in the shape of two circles C_1 and C_2 which intersect each other, as shown in the diagram below (**not drawn to scale**).



The equations of C_1 and C_2 are $x^2 + y^2 + 4x + 6y - 3 = 0$ and $x^2 + y^2 + 4x + 2y - 7 = 0$ respectively.

A leg of the table is attached at EACH of the points Q and R where the circles intersect.

Determine the coordinates of the positions of the legs of the table. **[7 marks]**

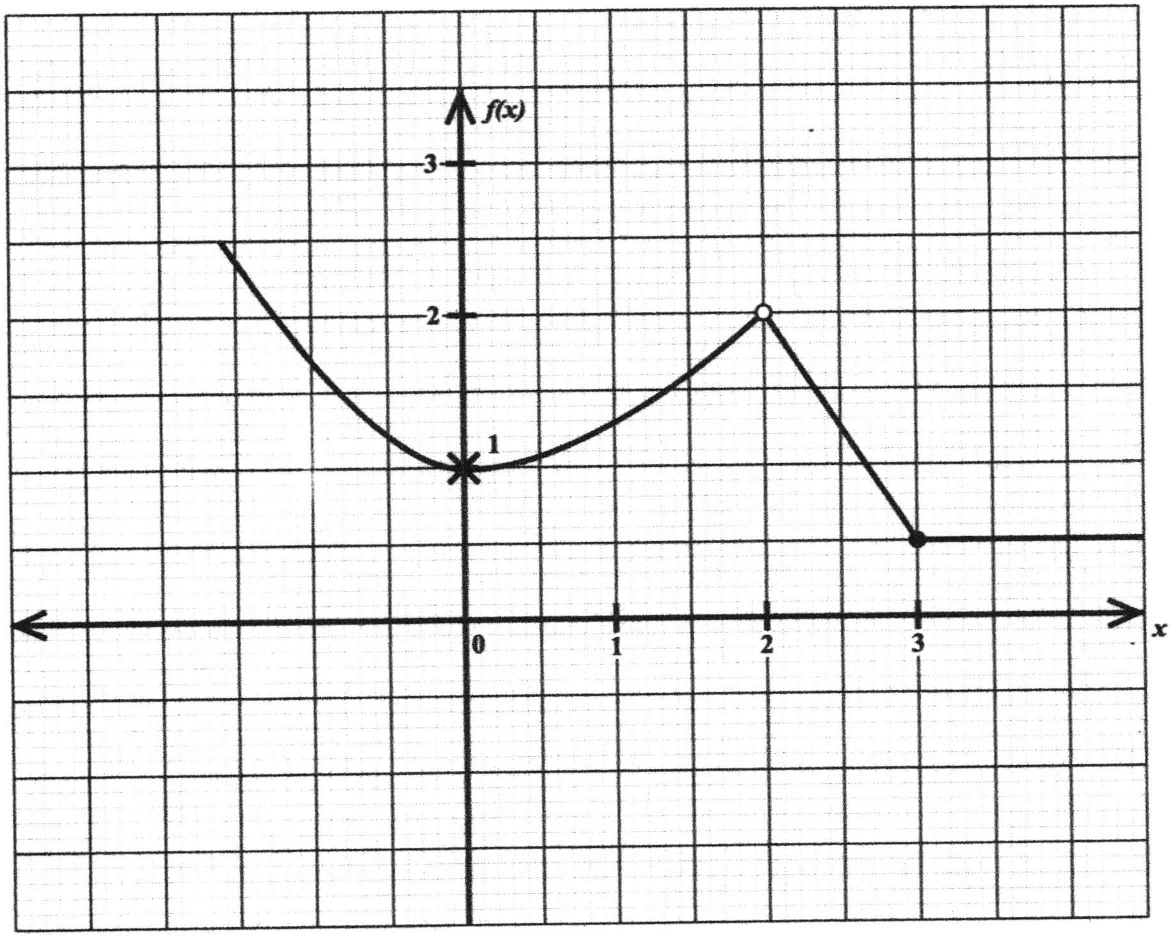
Total 20 marks

GO ON TO THE NEXT PAGE

SECTION C (Module 3)

Answer this question.

3. (a) The diagram below shows the graph of a function, $f(x)$.



(i) Determine for the function

a) $\lim_{x \to 0} f(x)$ **[1 mark]**

b) $\lim_{x \to 2} f(x)$. **[2 marks]**

(ii) State whether f is continuous at $x = 2$. **Justify your answer.** **[2 marks]**

GO ON TO THE NEXT PAGE

(b) Differentiate $f(x) = \frac{1}{\sqrt{2x}}$ from first principles. **[5 marks]**

(c) Find the x-coordinates of the maximum and minimum points of the curve

$$f(x) = 4x^3 + 7x^2 - 6x.$$ **[7 marks]**

(d) A water tank is made by rotating the curve with equation $\frac{x^2}{4} + \frac{y^2}{25} = 1$ about the x-axis between $x = 0$ and $x = 2$.

Find the volume of water that the tank can hold. **[3 marks]**

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.