PREVIEW UNIT 1 – TEST 3 (2014)

1. (a) Find

(i)

$$\lim_{x\to 2}\frac{x^3-64}{x^2-3x-4}$$

$$\left[\frac{28}{3}\right]$$

(ii)

$$\lim_{x\to 0}\frac{\sin 5x}{\sin 2x}$$

$$\left[\frac{5}{2}\right]$$

(iii) the value(s) of x for which $f(x)=\frac{|x-2|}{18-2x^2}$ is discontinuous. $[\pm 3]$

(b) The function f on $\mathbb{R}$ is defined by

$$f(x)=\begin{cases}x^2-4, & x<2\\ 4x-8, & x\geq 2\end{cases}$$

Determine

(i) $f(2)$

(ii) $\lim_{x\to 2^+} f(x)$

(iii) $\lim_{x\to 2^-} f(x)$ $[0, 0, 0]$

(c) (i) Given that $f(x)=x^3$, show that $f(x+h)=x^3+3x^2h+3xh^2+h^3$.

(ii) Hence differentiate $f(x)=\frac{1}{x^3}$ from first principles. $\left[f'(x)=-\frac{3}{x^4}\right]$

2. (a) Determine $f'(x)$ for each of the following

(i) $f(x)=(3x-2)(x^3+1)^4$ $[3(x^3+1)^3(13x^3-8x^2+1)]$

(ii) $f(x)=\sin(x^3)-\cot(x+7)$ $[3x^2\cos(x^3)+\text{cosec}^2(x+7)]$

(b) The curve $y=x+\frac{4}{x}$ passes through the point $A(1,5)$.

Determine

(i) the equation of the normal to the curve at A. $\left[y=\frac{1}{3}x+\frac{14}{3}\right]$

(ii) the coordinates of the stationary points on the curve.

(iii) the nature of each stationary point. $[(2,4)\text{minimum and }(-2,-4)\text{maximum}]$

3. (a) The oscillations of a 'baby bouncy cradle' are modelled by the differential equation

$$\frac{dy}{dt}=\frac{300\sin 3t}{y}$$

where y cm is the height of the cradle above its base t seconds after the cradle begins to oscillate.

Given that the cradle is 10 cm above its base at time $t=\frac{\pi}{3}$ seconds, show that the particular solution of the differential equation is

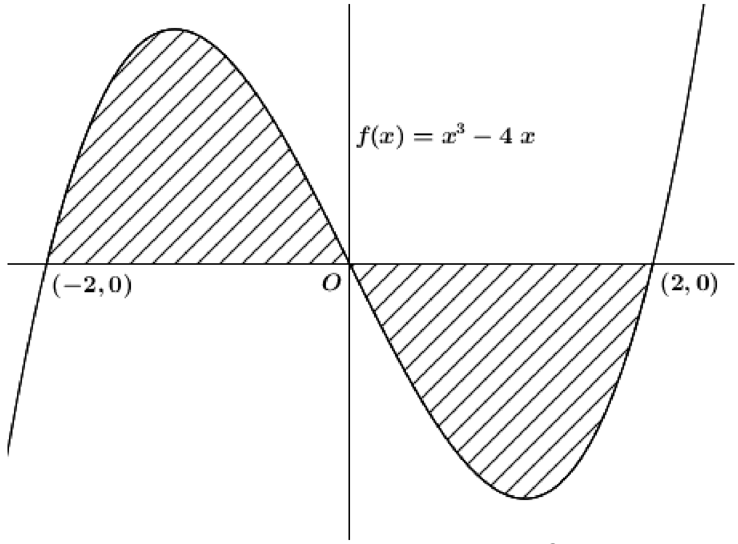
$$y^2=-200\sin 3t-100$$

(b) Using the substitution $u=2x^4-5$, evaluate

$$\int 2x^3(2x^4-5)^5\,dx$$

$$\left[\frac{(2x^4-5)^6}{24}+\text{c}\right]$$

(c)



The diagram above shows a portion of the graph of $f(x) = x^3 - 4x$.
Determine the area of the shaded region. [8 unit^2]