PURE MATHEMATICS
UNIT 1 – TEST 2 (PREVIEW)
1 hour 20 minutes

1. (i) Prove that

$$\frac{\cos 2\theta}{\sin 2\theta} + \frac{1}{\sin 2\theta} \equiv \cot\theta \qquad [4]$$

   (ii) Hence show that $\cot 15° = 2 + \sqrt{3}$ [3]

2. (i) Express $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

   (ii) Hence

   (a) Solve the equation $4\sin\theta - 3\cos\theta + 1 = 0$, giving all solutions for which $-180° < \theta < 180°$ [4]

   (b) Find the values of the positive constants k and c such that

$$-37 \leq k(4\sin\theta - 3\cos\theta) + c \leq 43$$

   for all values of θ. [4]

3. The circle C has equation $x^2 + y^2 - 12x - 8y + 44 = 0$.

   (a) Find the coordinates of the centre and the radius of C. [3]

   (b) Find the exact distance of the centre of C from the origin. [2]

   The point A lies on C and the tangent to C at A passes through the point $B(0, 2)$.

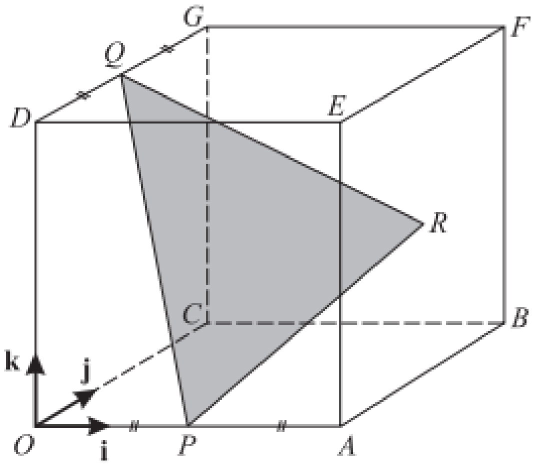
   (c) Show that $|AB| = 4\sqrt{2}$. [3]

4. Given the following equation $16x^2 + y^2 = 64$

   (a) Find the x and y intercepts of the graph of the equation. [4]

   (b) Find the length of the major and minor axes. [2]

   (c) Sketch the graph of the equation. [2]



5. The diagram shows a cube $OABCDEFG$ in which the length of each side is 8 units. The unit vectors $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ are parallel to $\overrightarrow{OA}, \overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The mid-points of OA and DG are P and Q respectively and R is the centre of the square face $ABFE$.

   (i) Express each of the vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ in terms of $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$. [3]

   (ii) Use a scalar product to find angle QPR. [4]

   (iii) Find the perimeter of triangle PQR, giving your answer correct to 1 decimal place. [3]

6. The position vectors of points A and B relative to an origin O are $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively. The position vectors of points C and D relative to O are $3\boldsymbol{a}$ and 2b respectively. It is given that

$$a = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \text{ and } b = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}$$

   (i) Find the unit vector in the direction of $\overrightarrow{CD}$. [3]

   (ii) The point E is the mid-point of CD. Find angle EOD. [6]

7. The line L passes through the points P and Q with position vectors $3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{2k}$ and $-\boldsymbol{j} + 4\boldsymbol{k}$ respectively.

   (i) Find the equation of L, giving your answer in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. [2]

   (ii) Show that the point S with position vector $9\boldsymbol{i} + 5\boldsymbol{j} - \boldsymbol{2k}$ lies on L, and find the ratio of the length of PS to the length of QS. [3]

   (iii) Find the acute angle between L and a line with direction vector $\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{2k}$, giving your answer correct to the nearest degree. [3]