PREVIEW UNIT 1 TEST 2 (2014)

1. (a) Determine the Cartesian equation for the curve defined parametrically by

$$x = \sin t \quad y = \cot t$$

(b) The circle C_1 has equation $x^2 + y^2 + 6x - 8y = 0$. Determine

(i) the centre and radius of C_1.

(ii) the exact length of the tangent from the point $A(3, -4)$.

The circle C_2 with equation $x^2 + y^2 + 4x - 4y = 12$ intersects C_1 at P and Q.

(iii) Determine the coordinates of P and Q.

Ans: (a) $y^2 = \frac{1-x^2}{x^2}$ (b)(i) $C(-3, 4), r = 5$ (ii) $5\sqrt{3}$ (iii) $(-6, 0)(2,4)$

2. (a) Prove that $\frac{\csc x}{\tan x} = \cot x \csc x$

(b) Find the general solutions of the equation

$$2\sin^2\theta - \cos\theta = 1$$

(c) (i) Express $f(\theta) = 6\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0 \le \alpha < \frac{\pi}{2}$.

(ii) Hence, state the maximum value of $f(\theta)$ and the value of θ for which this maximum occurs.

Ans: (b) $\frac{\pi}{3} + 2n\pi; \frac{5\pi}{3} + 2n\pi; (2n+1)\pi, n \in \mathbb{Z}$ (c) (i) $\sqrt{45}\cos(\theta - 0.464^c)$ (ii) $f(\theta)_{max} = \sqrt{45}$, $\theta = 0.464^c$

3. The position vectors of the points A, B, C are given by

$$a = 4\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k},\ b = \boldsymbol{i} + 2\boldsymbol{k},\ c = 2\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}$$

(a) Determine

(i) the vectors

(a) $\overrightarrow{AB}$,

(b) $\overrightarrow{BC}$

(ii) the equation of the line, l, which passes through the points A and B.

(iii) the angle between a and c.

(b) (i) Show that the vector $-\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ is perpendicular to the plane through the points A, B and C.

(ii) Hence, determine the equation of the plane in the form $r.n = d$.

Ans: (a) (i) $\overrightarrow{AB} = \begin{pmatrix} -3 \\ -3 \\ -3 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$ (ii) $l = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -3 \\ -3 \end{pmatrix}$ (iii) $18.43°$ (b) (ii) $r.\begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} = 5$