UNIT 1 MODULE 2 TEST

1 hour 10 minutes

Instructions: Answer ALL questions, giving your answers to 3 significant figures

1. If $90° < \alpha < 180°$ with $\sin\alpha = \frac{5}{13}$ and $0° < \beta < 90°$ with $\cos\beta = \frac{8}{17}$, find the value in exact form of

   (a) $\cos\alpha$ [2]

   (b) $\sin\beta$ [2]

   (c) $\cot\alpha$ [2]

   (d) $\sin(\alpha - \beta)$ [3]

   (e) $\cos\left(\frac{\alpha}{2}\right)$ [3]

2. Find the general solution of the equation

   $$\tan 3x = \sqrt{3}$$

   giving your answers in terms of π. [4]

3. By expressing $\tan3\theta$ as $\tan(2\theta + \theta)$ show that $tan3\theta = \frac{3tan\theta - tan^3\theta}{1-3tan^2\theta}$ [5]

4. (a) Express $5\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [5]

   (b) Hence state the maximum value of $5\sin\theta - 2\cos\theta + 3$ and the value of θ for which it occurs [4]

   (c) Find the values of x for which $5\sin\theta - 2\cos\theta = 4$ for $0° < \theta < 360°$ [4]

5. The vectors **m**, **n** and **p** are given by $\mathbf{m} = \begin{pmatrix} r \\ 3 \\ -6 \end{pmatrix}$, $\mathbf{n} = \begin{pmatrix} 4 \\ s \\ 2 \end{pmatrix}$ and $\mathbf{p} = \begin{pmatrix} 16 \\ 3 \\ t \end{pmatrix}$.

   (a) If $2\mathbf{m} + 3\mathbf{n} = \mathbf{p}$ find the value of r, of s and of t. [4]

   (b) Find a unit vector parallel to **m**. [3]

6. In Ms Murray's Chemistry class it was observed that three corners of a crystal were located at the points A(3, 2, 4), B(2, 4, 7) and C(4, 5, 9) relative to an origin O.

(a) Find, in column form, the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [3]

(b) Calculate the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [4]

(c) Show that $\begin{pmatrix} 1 \\ 8 \\ -5 \end{pmatrix}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [2]

7. The parametric equations for x and y are defined by

$$x = t - 2 \text{ and } y = 2t^2 + 1$$

(a) Find the Cartesian equation which connects x and y. [2]

(b) Hence sketch the graph of the relationship for $-2 < t < 4$. [3]

8. (a) Show that the equation of the tangent to the circle $x^2 + y^2 = 5$ at the point (–2, 1) is $y = 2x + 5$. [4]

This tangent intersects the circle $x^2 + y^2 - 6x - 12y + 35 = 0$ at points P and Q.

(b) Calculate the coordinates of P and Q. [7]

(c) Show that the tangents to the second circle at P and Q are perpendicular to each other. [4]