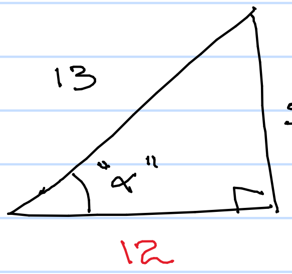
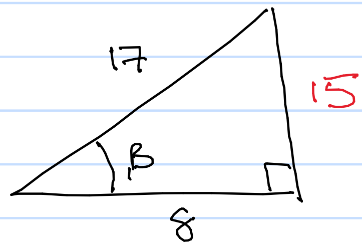
UNIT 1 MODULE 2 PRACTICE TEST 1

1)





(a) If $\sin\alpha = \frac{5}{13}$ and $90 < \alpha < 180$

$$\Rightarrow \cos\alpha = -\frac{12}{13}$$

(b) $\sin\beta = \frac{15}{17}$

(c) $\cot\alpha = -\frac{1}{\frac{5}{12}} = -\frac{12}{5}$

(d)
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$= \frac{5}{13}\cdot\frac{8}{17} - \left(\frac{-12}{13}\right)\frac{15}{17}$$
$$= \frac{220}{221}$$

(e)
$$\cos\left(2\left(\frac{\alpha}{2}\right)\right) = 2\cos^2\left(\frac{\alpha}{2}\right) - 1$$
$$\cos^2\left(\frac{\alpha}{2}\right) = \frac{\cos\alpha+1}{2}$$
$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{\cos\alpha+1}{2}} = \sqrt{\frac{1-\frac{12}{13}}{2}}$$
$$= \sqrt{\frac{1}{26}}$$

2. $\tan 3x = \sqrt{3}$

$$\alpha = \tan^{-1}\sqrt{3} \Rightarrow \frac{\pi}{3}$$

$$3x = n\pi + \frac{\pi}{3}$$

$$x = \frac{n\pi}{3} + \frac{\pi}{9}$$

3. $\tan(3\theta) = \tan(2\theta + \theta)$

$$= \frac{\tan 2\theta + \tan\theta}{1 - \tan\theta\tan 2\theta}$$

$$= \frac{\dfrac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1 - \tan\theta\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)} \times \frac{1-\tan^2\theta}{1-\tan^2\theta}$$

$$= \frac{2\tan\theta + \tan\theta - \tan^3\theta}{1 - \tan^2\theta - 2\tan^2\theta}$$

$$= \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^3\theta}$$

4. $5\sin\theta - 2\cos\theta = R\sin(\theta - \alpha)$

$$R = \sqrt{5^2 + 2^2} = \sqrt{29}$$

$$\alpha = \tan^{-1}\tfrac{2}{5} = 21.8^\circ$$

$$5\sin\theta - 2\cos\theta = \sqrt{29}\sin(\theta - 21.8)$$

(b) Max value $= \sqrt{29} + 3$

occurs at $\theta = 90 + 21.8 = 111.8^\circ$

(c) $\sin(\theta - 21.8) = \frac{4}{\sqrt{29}} \Rightarrow \theta - 21.8 = 48^\circ, 132^\circ$

$\theta = 69.8^\circ, 153.8^\circ$

5. $2\underset{\sim}{m} + 3\underset{\sim}{n} = \underset{\sim}{p}$

$$2\begin{pmatrix} r \\ 3 \\ -6 \end{pmatrix} + 3\begin{pmatrix} 4 \\ s \\ 2 \end{pmatrix} = \begin{pmatrix} 16 \\ 3 \\ t \end{pmatrix}$$

$2 + 12 = 16 \Rightarrow r = 2$

$6 + 3s = 3 \Rightarrow s = -1$

$-12 + 6 = t \Rightarrow t = -6$

(b) $m = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$, $|m| = \sqrt{2^2 + 3^2 + (-6)^2}$

$= \sqrt{49} = 7$

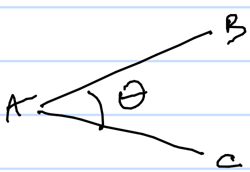
unit vector parallel to $\underset{\sim}{m}$

$$= \frac{1}{7}\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$$

6. $\overline{AB} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 5 \\ 9 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$

(b)



$\overline{AB} \cdot \overline{AC} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} = -1 + 6 + 15 = 20$

$|AB| = \sqrt{1 + 4 + 9} = \sqrt{14}$

$|AC| = \sqrt{1 + 9 + 25} = \sqrt{35}$

$\cos\theta = \dfrac{20}{\sqrt{14}\sqrt{35}} \Rightarrow \theta = 25.4^{\circ}$

6 (c) $\begin{pmatrix} 1 \\ 8 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = -1 + 16 - 15 = 0$

so $\begin{pmatrix} 1 \\ 8 \\ -5 \end{pmatrix}$ is $\perp$ to $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ (AB)

$\begin{pmatrix} 1 \\ 8 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} = 1 + 24 - 25 = 0$

so $\begin{pmatrix} 1 \\ 8 \\ -5 \end{pmatrix}$ is $\perp$ to AC: $\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$

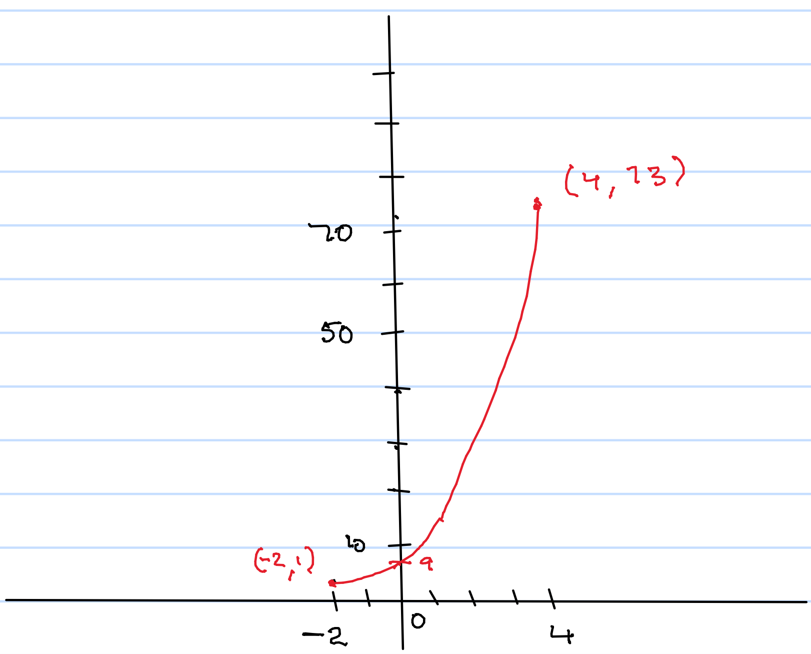
7 (a) $x = t - 2 \implies t = x + 2$

$y = 2t^2 + 1$

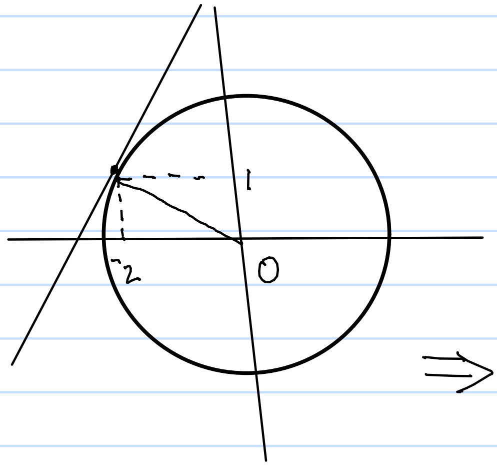
$y = 2(x+2)^2 + 1$

$y = 2x^2 + 8x + 9$

(b) $y = 2x^2 + 8x + 9 = 2(x+2)^2 + 1$



8 (a)



$m_{\text{radius}} = -\frac{1}{2}$

m tangent $= 2$

equation of tangent

$\Rightarrow y - 1 = 2(x+2)$

$y = 2x + 5$

(b) $x^2 + y^2 - 6x + 12y + 35 = 0$

$x^2 + (2x+5)^2 - 6x - 12(2x+5) + 35 = 0$

$5x^2 - 10x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0$ or $x = 2$

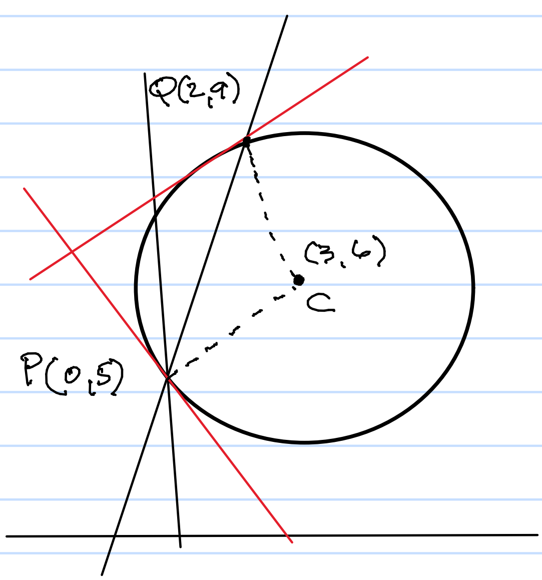
when $x = 0$ $y = 5$ $\Rightarrow$ $P(0,5)$

when $x = 2$ $y = 9$ $\Rightarrow$ $Q(2,9)$

(c) $x^2 - 6x + y^2 - 12y + 35 = 0$

$(x-3)^2 + (y-6)^2 - 10 = 0$

so centre of circle is $(3,6)$



$m_{CP} = \frac{1}{3} \Rightarrow m_{\text{tangent}} = -3$

$m_{CQ} = -3 \Rightarrow m_{\text{tangent}} = \frac{1}{3}$

so since $-3 \times \frac{1}{3} = -1$

tangents are $\perp$ to each