Harrison College – Barbados

Solution and Marking Key

CAPE 2014: Unit 1 Test 3

1. (a) (i)	$\lim_{x\to 2}\frac{x^3-8}{x^2-x-2}$ $=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x+1)}$ $=\lim_{x\to 2}\frac{x^2+2x+4}{x+1}$ $=\frac{2^2+2(2)+4}{2+1}$ $=4$	Factoring x^3-8 Factoring x^2-x-2 Cancelling $x-2$ C.A.O	1 1 1 1
(ii)	$\lim_{x\to 0}\frac{\sin 2x}{\sin 5x}$ $=\lim_{x\to 0}\sin 2x \div \lim_{x\to 0}\sin 5x$ $=\lim_{x\to 0}\frac{\sin 2x}{2x}\times 2x \div \lim_{x\to 0}\frac{\sin 5x}{5x}\times 5x$ $=\frac{2x}{5x}=\frac{2}{5}$	Splitting limit Correct use of $2x$ and $5x$ Use of $\lim_{x\to 0}\frac{\sin x}{x}=1$ C.A.O	1 1 1 1
(iii)	$f(x)=\frac{4}{x^2-16}$ is discontinuous when $x^2-16=0$ $x^2=16$ $x=\pm 4$	Equating denominator to zero $x=4$ $x=-4$	1 1 1
(b)(i)	$\lim_{x\to 3^+} f(x)=4(3)-6=6$ $\lim_{x\to 3^-} f(x)=(3)^2-2(3)+3=6$ $\lim_{x\to 3} f(x)=6$	Attempt at evaluating both left hand and right hand limit Correct value for left hand limit Correct value for right hand limit Conclusion	1 1 1 1
(ii)	$f(3)=4(3)-6=6$ $f(x)$ is continuous at $x=3$ since $\lim_{x\to 3} f(x)=f(3)$	Determining value of $f(3)$ Conclusion	1 1
(c)(i)	$f(x+h)=(x+h)^2$ $=x^2+2hx+x^2$	Replacing x with $x+h$ Correct expansion	1 1
(ii)	$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{\left(\frac{1}{x^2+2hx+h^2}-\frac{1}{x^2}\right)}{h}$ $=\lim_{h\to 0}\frac{x^2-(x^2+2hx+h^2)}{x^2(x^2+2hx+x^2)}\times\frac{1}{h}$ $=\lim_{h\to 0}\frac{h(-2x-h)}{x^2(x^2+2hx+h^2)}\times\frac{1}{h}$ $=\lim_{h\to 0}\frac{-2x-h}{x^2(x^2+2hx+h^2)}$	Correct form for first principles Sub. expressions for $f(x+h)$ and $f(x)$ Expression for $\frac{1}{f(x+h)}-\frac{1}{f(x)}$ Cancelling h Sub. $h=0$	1 1 1 1 1

	$= \frac{-2x - 0}{x^2(x^2 + 2(0)x + 0^2)}$ $= -\frac{2x}{x^4}$ $= -\frac{2}{x^3}$	Simplifying derivative	1
2.(a)(i)	$f(x) = (2x - 1)(x^2 + 5)^3$ $f'(x) = 2(x^2 + 5)^3 + (2x - 1)3(x^2 + 5)^2(2x)$	Obtaining $2(x^2 + 5)^3$ Obtaining $(2x - 1)(x^2 + 5)^2$ Multiplying $(2x - 1)(x^2 + 5)^2$ by $6x$	1 1 1
(ii)	$g(x) = \cos(x^2) - \tan(x - 3)$ $= -\sin(x^2) \times (2x) - \sec^2(x - 3)$ $= -2x\sin(x^2) - \sec^2(x - 3)$	Obtaining $-\sin(x^2)$ Multiplying $-\sin(x^2)$ by $2x$ Differentiating $\tan(x - 3)$	1 1 1
(b)(i)	$y = 2x + \frac{8}{x} = 2x + 8x^{-1}$ $\frac{dy}{dx} = 2 - \frac{8}{x^2}$ $\frac{dy}{dx_{x=4}} = 2 - \frac{8}{4^2} = \frac{3}{2}$ Gradient of normal is $-\frac{2}{3}$ $y = mx + c$ $10 = -\frac{2}{3}(4) + c$ $\frac{38}{3} = c$ $y = -\frac{2}{3}x + \frac{38}{3}$	Correctly differentiating $2x + \frac{8}{x}$ Sub. $x = 4$ into his $\frac{dy}{dx}$ and obtaining value of gradient of tangent Correctly obtaining gradient of his normal Sub. $m = -\frac{2}{3}, x = 4, y = 10$ into $y = mx + c$ Correct equation	1 1 1 1 1
(ii)	Stationary points occur when $\frac{dy}{dx} = 0$ $2 - \frac{8}{x^2} = 0$ $2 = \frac{8}{x^2}$ $x^2 = 4$ $x = \pm 2$ $y = 2(2) + \frac{8}{2} = 8$ $(2, 8)$ $y = 2(-2) + \frac{8}{-2} = -8$ $(-2, -8)$	Equating $\frac{dy}{dx} = 0$ Solving for both values of x One corresponding value of y Second corresponding value of y	1 1 1 1
(iii)	$\frac{d^2y}{dx^2} = \frac{16}{x^3}$ When $x = 2$ $\frac{16}{2^3} = 2$ therefore $(2, 8)$ is a minimum When $x = -2$ $\frac{16}{(-2)^3} = -2$ therefore $(-2, -8)$ is a maximum	Obtaining second derivative Correct conclusion based on $\frac{d^2y}{dx^2}$ Correct conclusion based on $\frac{d^2y}{dx^2}$	1 1 1
3. (a)	$\frac{dy}{dt} = \frac{150\cos 2t}{y}$		

	$y\,dy = 150\cos 2t\;dt$ $\int y\;dy = \int 150\cos 2t\;dt$ $\frac{y^2}{2} = \frac{150\sin 2t}{2} + c$ $y^2 = 150\sin 2t + c$ $(20)^2 = 150\sin\left(2\left(\frac{\pi}{4}\right)\right) + c$ $400 = 150(1) + c$ $250 = c$ $y^2 = 150\sin 2t + 250$	Separating variables Integrating $y\,dy$ Integrating $150\cos 2t\;dt$ Sub. $y = 20$ and $t = \frac{\pi}{4}$ Correct value of c	1 1 1 1 1
(b)	$u = 2x^3 - 5$ $\frac{du}{dx} = 6x^2$ $\frac{du}{6x^2} = dx$ $\int (6x)^2(2x^3-5)^4\;dx = \int 6(u)^4\;du$ $= \frac{6u^5}{5} + c$ $= \frac{6(2x^3-5)^5}{5} + c$	Obtaining expression for dx Sub. $u = 2x^3 - 5$ and $dx = \frac{du}{6x^2}$ Simplifying integral in terms of u Integrating with respect to u C.A.O	1 1 1 1 1
(c)	$\int_0^1 (x^3 - 2x^2 - x + 2)\;dx$ $= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x\right]$ $= \left(\left[\frac{1^4}{4} - \frac{2(1)^3}{3} - \frac{(1)^2}{2} + 2(1)\right] - \left[\frac{0^4}{4} - \frac{2(0)^3}{3} - \frac{(0)^2}{2} + 2(0)\right]\right)$ $= \left[\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right]$ $= \frac{13}{12}$ $\int_1^2 (x^3 - 2x^2 - x + 2)\;dx$ $= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x\right]$ $= \left(\left[\frac{2^4}{4} - \frac{2(2)^3}{3} - \frac{(2)^2}{2} + 2(2)\right] - \left[\frac{1^4}{4} - \frac{2(1)^3}{3} - \frac{(1)^2}{2} + 2(1)\right]\right)$ $= \left(\left[4 - \frac{16}{3} - 2 + 4\right] - \left[\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right]\right)$ $= -\left(\frac{2}{3} - \frac{13}{12}\right)$ $= \frac{3}{2}$	Use of the correct limits Integrating $x^3 - 2x^2 - x + 2$ Sub. limits into integral Correct value of definite integral Sub. limits into integral Ensuring that value is positive Adding two areas together	1 1 1 1 1 1 1

End of Marking Key