HARRISON COLLEGE INTERNAL EXAMINATIONS 2011 : PURE MATHEMATICS [UNIT1 – TEST 3]

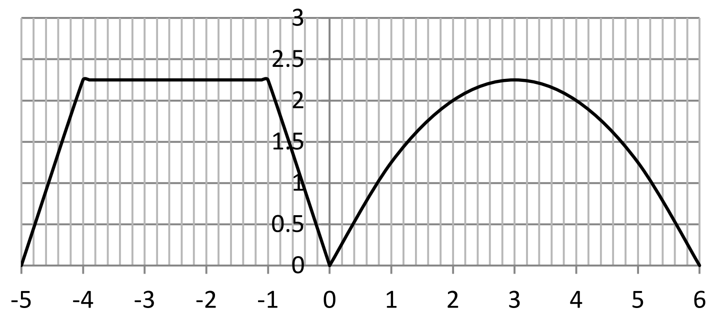
SOLUTIONS AND MARK SCHEME

1. (a) (i) $\lim_{x\to 0}\frac{3sin2x}{x}$ = $\lim_{x\to 0}\frac{6sin2x}{2x}$ = 6 . 1 = 6 [2]

(ii) $\lim_{x\to -2}[4f(x)] = 12 \Rightarrow \lim_{x\to -2}f(x) = \frac{12}{4} = 3$

So $\lim_{x\to -2}[f(x) - 2x] = 3 - 2(-2) = 7$ [3]

(b) (i)



[4]

(ii) $\lim_{x\to -1^+} f(x) = -\frac{9}{4} \times (-1) = \frac{9}{4}$ [2]

(iii) f is **NOT** differentiable at $x = -1$ as there is a sudden change in gradient at that point in the graph . [2]

(c) $\frac{dy}{dx} = \lim_{h\to 0}[\frac{f(x+h) - f(x)}{h}]$ therefore If $y = \sqrt{2x}$

$$\frac{dy}{dx} = \lim_{h\to 0}[\frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}] = \lim_{h\to 0}\frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}\cdot\frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}}$$

$$= \lim_{h\to 0}\frac{2(x+h) - 2x}{h(\sqrt{2(x+h)} + \sqrt{2x})} = \lim_{h\to 0}\frac{2h}{h(\sqrt{2(x+h)} + \sqrt{2x})}$$

$$= \lim_{h\to 0}\frac{2}{\sqrt{2(x+h)} + \sqrt{2x}} = \frac{2}{2\sqrt{2x}} = \frac{1}{\sqrt{2x}}$$ [4]

2. (a) $y = 2x^3 - 4x^2 + 9$ at $x = 3$ $y = 2(3^3) - 4(3^2) + 9 = 27$

$\frac{dy}{dx} = 6x^2 - 8x$ so at $x = 3$, $\frac{dy}{dx} = 6(3^2) - 8(3) = 30$

So equation of normal at (3, 27): $(y - 27) = -\frac{1}{30}(x - 3)$

or $y = -\frac{1}{30}x + 27\frac{1}{10}$ [6]

(b) (i) $y = x\sqrt{x^2 - 1} = x(x^2 - 1)^{\frac{1}{2}}$

$$\frac{dy}{dx} = x \cdot \frac{1}{2} \cdot (x^2 - 1)^{-\frac{1}{2}} \cdot (2x) + (x^2 - 1)^{\frac{1}{2}} \cdot (1)$$

$$= x^2(x^2 - 1)^{-\frac{1}{2}} + (x^2 - 1)^{\frac{1}{2}}$$ [4]

(ii) $y = \frac{\sin^2 x}{x^2}$

$$\frac{dy}{dx} = \frac{x^2(2\sin(x) \cdot \cos(x)) - \sin^2 x \cdot (2x)}{x^4}$$ [4]

3. $V = 2\pi r^2$ $\Rightarrow$ $\frac{dV}{dr} = 4\pi r$ when $r = 30$ $\frac{dV}{dr} = 4 \times \pi \times 30 = 120\pi$

$\frac{dV}{dt} = 10 \ mm^3/min$

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ so $\frac{dr}{dt}\ (when\ r = 30) = \frac{1}{120\pi} \times 10 = \frac{1}{12\pi}\ mm/min$ [4]

4. (i) $f(x) = x - \frac{6}{x} + \frac{9}{x^3} \Rightarrow f^{/}(x) = 1 + \frac{6}{x^2} - \frac{27}{x^4} = 0$

$$x^4 + 6x^2 - 27 = 0$$

$$(x^2 + 9)(x^2 - 3) = 0$$

$x^2 = -9$ [no real solutions]

$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$ [7]

(ii) $f^{||}(x) = -\frac{12}{x^3} + \frac{108}{x^5}$

When $x = +\sqrt{3}$ $f^{||}(x) = -\frac{12}{\sqrt{3}^3} + \frac{108}{\sqrt{3}^5} = +4.62$ so point is a minimum.

Thus at $x = -\sqrt{3}$ point is a maximum. [3]

5. (i) $\int [\,tan^2x + \cos 3x\,]\,dx = \int [\,sec^2x - 1 + cos\,3x\,]\,dx$

$= tanx - x + \frac{\sin 3x}{3} + constant$ [4]

(ii) $u = x^2 - 1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$\int \frac{3x}{(x^2 - 1)^2}\,dx = \int \frac{3}{2\,u^2}\,du = -\frac{3}{2}\,u^{-1} + constant$

$= -\frac{3}{2(x^2 - 1)} + constant$ [4]

6. $V = \pi \int_{-1}^{1} ({y_1}^2 - {y_2}^2)\ dx \quad \Rightarrow \quad V = \pi \int_{-1}^{1} [(4 - 4x^2) - (1 - x^2)]\ dx$

$$V = \pi \int_{-1}^{1} [(3 - 3x^2)\ ]\ dx$$

$$V = \pi\, [3x - x^3\,]_{-1}^{1}$$

$V = \pi[(3-1) - (-3+1)] = 4\pi \ \ units^3$ [7]

[TOTAL = 60 marks]