HC SBA UNIT 1 TEST 2 (2013)

1(a) $x = 2t \Rightarrow t = \frac{x}{2}$

$$y = \frac{7x}{2} - 5\left(\frac{x}{2}\right)^2$$

$$y = \frac{7x}{2} - \frac{5x^2}{4}$$

(b)(i) $x = -2 + 3\sin t$

maximum value of x
$= -2 + 3 = 1$

$y = 4 + 2\cos t$

minimum value of y
$= 4 - 2 = 2$

(ii) $\sin t = \frac{x+2}{3} \Rightarrow \sin^2 t = \left(\frac{x+2}{3}\right)^2$

$\cos t = \frac{y-2}{4} \Rightarrow \cos^2 t = \left(\frac{y-2}{4}\right)^2$

$\sin^2 t + \cos^2 t = 1$

$$\frac{(x+2)^2}{9} + \frac{(y-2)^2}{16} = 1$$

(c) (i) $r = \sqrt{3^2 + 4^2} = 5$

(ii) $(x-2)^2 + (y-1)^2 = 5$

(iii) $m_{radius} = \frac{4}{3}$

$\therefore m_{tangent} = -\frac{3}{4}$

$y - 5 = -\frac{3}{4}(x-5)$

1 (iv) $D_1: (x-2)^2 + (y-1)^2 - 25 = 0$

$$x^2 + y^2 - 4x - 2y - 20 = 0$$

$D_2: x^2 + y^2 - 10x + 4y - 8 = 0$

$$6x - 6y - 12 = 0$$
$$x - y - 2 = 0$$
$$y = x - 2$$

$$x^2 + (x-2)^2 - 4x - 2(x-2) - 20 = 0$$
$$2x^2 - 10x - 12 = 0$$
$$x^2 - 5x - 6 = 0$$
$$(x-6)(x+1) = 0$$

$x = 6 \quad x = -1$

$y = 4 \quad y = -3$

$H(6, 4)$ $\overset{4}{\underset{1}{\vdots}}$ $G(-1, -3)$

2 (i) $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -4 \end{pmatrix}$

(ii) $\overrightarrow{AB}$ = 4 x direction vector of l

$$= 4 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -4 \end{pmatrix}$$

(iii) If $\lambda = 2$

$$\begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \\ -1 \end{pmatrix}$$ which is $\overrightarrow{OD}$

2(b) direction vector of m:

$$\begin{pmatrix} 9 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$$

vector equation of m

$$r = \begin{pmatrix} 7 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$$

(iii) $r = \begin{pmatrix} 7 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$

$$\vec{AC} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

$$AC \cdot m = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} = 4$$

$$|AC| = \sqrt{(-2)^2 + 1} = \sqrt{5}$$

$$\left| \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} \right| = \sqrt{2^2 + 4^2} = \sqrt{20}$$

$$\cos\theta = \frac{4}{\sqrt{5}\sqrt{20}} = 0.4$$

$$\theta = \cos^{-1} 0.4 = 66.4^\circ$$

(c) $r \cdot n = a \cdot n$

$$r \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 2 + 3 + 8 = 13$$

$$r \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 13$$

3 (a) $I = \dfrac{k \tan\theta}{d^2 \sec\theta}$

$= \dfrac{k}{d^2} \dfrac{\sin\theta}{\cos\theta} \cdot \cos\theta$

$= \dfrac{k}{d^2} \sin\theta$

(b) $\sqrt{3}\sin x + \cos x = R\sin(x+\alpha)$

$R = \sqrt{3^2 + 1^2} = \sqrt{4} = 2$

$\tan\alpha = \dfrac{1}{\sqrt{3}} \Rightarrow \alpha = \dfrac{\pi}{6}$

$\sqrt{3}\sin x + \cos x = 2\sin\left(x + \dfrac{\pi}{6}\right)$

b (i) $\sqrt{3}\sec x + \operatorname{cosec} x = 4$

$\dfrac{\sqrt{3}}{\cos x} + \dfrac{1}{\sin x} = 4$

$\dfrac{\sqrt{3}\sin x + \cos x}{\sin x \cos x} = 4$

$\sqrt{3}\sin x + \cos x = 4\sin x \cos x$

$\sqrt{3}\sin x + \cos x = 2(2\sin x \cos x)$

$\sqrt{3}\sin x + \cos x = 2\sin 2x$

(iii) from b(i) $\sqrt{3}\sin x + \cos x = 2\sin\left(x + \dfrac{\pi}{6}\right)$

$2\sin\left(x + \dfrac{\pi}{6}\right) = 2\sin 2x$

$\sin 2x - \sin\left(x + \dfrac{\pi}{6}\right) = 0$

3(iv) $\sin 2x - \sin\left(x + \frac{\pi}{6}\right) = 0$

$$2\cos\left(\frac{3x + \frac{\pi}{6}}{2}\right) \sin\left(\frac{x - \frac{\pi}{6}}{2}\right) = 0$$

$$\frac{3x + \frac{\pi}{6}}{2} = \frac{\pi}{2} \quad \text{and} \quad \frac{x - \frac{\pi}{6}}{2} = 0, \pi$$

$$x = \frac{\pi - \frac{\pi}{6}}{3} \qquad x = \frac{\pi}{6}$$

$$= \frac{5\pi}{18} \qquad x = 2\pi + \frac{\pi}{6}$$

$$x = \frac{13\pi}{6}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{18}$$