HC SBA TEST. UNIT 1 TEST 2 (2013)

**UNIT 1 – TEST 2**
**1 hour 30 minutes**

This examination paper consists of 2 printed pages.

This paper consists of 3 questions.

The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

(i) Write your name clearly on each sheet of paper used.

(ii) Answer **ALL** questions.

(iii) Number your questions identically as they appear on the question paper and do **NOT write your solutions to different questions** beside each other.

(iv) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

(i) Mathematical formulae

(ii) Scientific calculator (non-programmable, non-graphical)

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1. (a) The path of a projectile can be modelled by the parametric equations

$$x = 2t; \quad y = 7t - 5t^2$$

Write the path of the projectile in the form $y = ax^2 + bx$ where $a$ and $b$ are real numbers. [3]

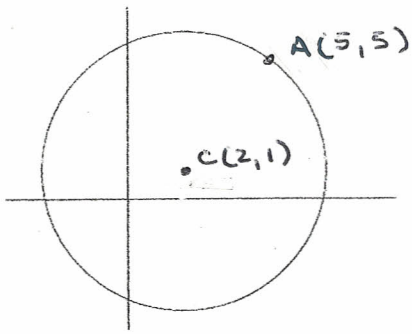
(b) A point moves so that at time $t$ the distances from the coordinate axes are given by

$$x = -2 + 3\sin t; \quad y = 4 + 2\cos t$$

(i) Determine the maximum value of $x$ and the minimum value of $y$. [2]

(ii) Determine the Cartesian equation of the curve traced by the point. [3]

(c)




In the diagram above, not drawn to scale, $C(2, 1)$ is the centre of the circle, $D_1$, and $A(5, 5)$ is a point on the circle. Determine

i. the length of the radius of the circle. [2]

ii. the equation of the circle in the form $(x - h)^2 + (y - k)^2 = r^2$. [2]

iii. the equation of the tangent to the circle at the point $A$. [4]

Another circle, $D_2$, with equation $x^2 + y^2 - 10x + 4y = 8$ intersects $D_1$ at $H$ and $G$.

iv. Determine the coordinates of $H$ and $G$. [8]

2. The quadrilateral $ABCD$ has vertices $A(1,3,4)$, $B(5,3,0)$, $C(1,1,5)$ and $D(7,2,-1)$. The line $L$ has vector equation $r = \begin{pmatrix}5\\2\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\0\\-1\end{pmatrix}$.

(a) (i) Find the vector $\overrightarrow{AB}$. [2]

(ii) Show that the line $AB$ is parallel to $L$. [2]

(iii) Verify that $D$ lies on $L$. [2]

(b) The line $M$ passes through $D(7,2,-1)$ and $E(9,2,3)$.

(i) Find the vector equation of $M$. [2]

(ii) Find the angle between $M$ and $AC$. [5]

(c) Given that the vector $2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ is perpendicular to the plane $ABCD$, determine, in the form $r.n = d$, the equation of the plane $ABCD$. [3]

3. (a) The intensity of a bright spotlight on a certain point on a stage is given by $I = \frac{k\tan\theta}{d^2\sec\theta}$. In the formula, $k$ is a constant and $d$ is the distance from the spotlight to that point of the stage at which $I$ is measured. Show that $I = \frac{k\sin\theta}{d^2}$. [3]

(b) (i) Express $\sqrt{3}\sin x + \cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [4]

(ii) Show that the equation $\sqrt{3}\sec x + \csc x = 4$ can be written in the form

$$\sqrt{3}\sin x + \cos x = 2\sin 2x$$

[3]

(iii) Deduce from parts (a) and (b) that $\sqrt{3}\sec x + \csc x = 4$ can be written in the form

$$\sin 2x - \sin\left(x + \frac{\pi}{6}\right) = 0$$

[2]

(iv) Hence, using the identity $\sin X - \sin Y = 2\cos\left(\frac{X+Y}{2}\right)\sin\left(\frac{X-Y}{2}\right)$, or otherwise, find the values of $x$ in the interval $0 \le x \le \pi$, for which $\sqrt{3}\sec x + \csc x = 4$. [8]

**END OF EXAMINATION**