PURE MATHEMATICS
UNIT 1 - TEST 3 (PREVIEW)

TIME: 1 Hour & 20 minutes

1. (a) Find $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$ [3]

(b) Find $\lim_{x \to 0} \frac{sin(5x)}{3x}$ [3]

(c) The function f on $\mathbb{R}$ is defined by

$$f(x) = \begin{cases} \frac{x^2+3x-10}{x-2}, & if\ x \neq 2 \\ kx+1, & if\ x = 2 \end{cases}$$

Find the value of the constant k that makes f continuous at $x = 2$. [5]

(d) Let $y = x^{-2}$. Using first principles, find $\frac{dy}{dx}$. [4]

TOTAL 15 marks

2. (a) Find $f'(x)$ when:

(i) $f(x) = \sqrt{(x^2-4)}$ [3]

(ii) $f(x) = \frac{3x}{\sin 2x}$ [3]

(b) A manufacturer wants to manufacture cylindrical aluminium cans with a volume of 2 000 cm^3. The cans are closed. Let R be the **internal** radius and h be the **internal** height of the tub.

(i) Express h in terms of R. [2]

(ii) Show that the internal surface area A cm^2 is given by

$$A = \frac{4000}{R} + 2\pi R^2$$ [3]

(iii) Hence determine the value of R which minimises the amount of material to be used. [3]

(c) A curve is defined by the parametric equations

$$x = 5t - 4 \qquad y = 1 - \frac{3}{t}$$

(i) Find $\frac{dy}{dx}$ in terms of t. [4]

(ii) Find the equation of the tangent to the given curve at the point where $t = 1$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [4]

TOTAL 22 marks

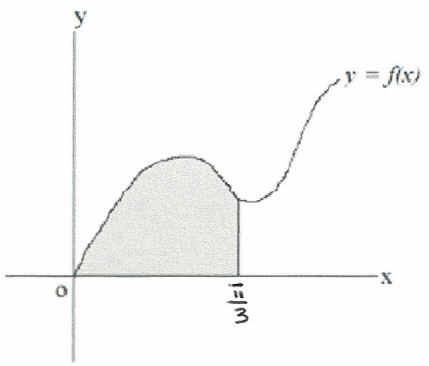
3. (a) The gradient of a curve is given by $\frac{dy}{dx} = (3x - 4)^{-2}$. The point $(0, 1)$ lies on the curve. Find the equation of the curve. [4]

(b) (i) Find $\int_0^1 \cos(2 - x)\, dx$. Give your answer to 2 decimal places. [4]

(ii) Using the substitution $u = x^2 - 1$, find

$$\int_1^2 2x(x^2 - 1)^3\, dx$$ [5]

(c) Find the shaded area in the diagram below which is bounded by the graph of $f(x) = \sin 3x + x$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{3}$.
Give your answer to 2 decimal places



[5]

(d) Solve the differential equation $\frac{dy}{dx} = 2\frac{x^3}{y}$ given that $y = 2$ when $x = 1$. [5]

TOTAL 23 marks