HARRISON COLLEGE INTERNAL EXAMINATION APRIL 2014
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 1 – TEST 3
1 hour 30 minutes

This examination paper consists of **2** printed pages.
This paper consists of 3 questions.
The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES

(i) Write your name clearly on each sheet of paper used
(ii) Answer **ALL** questions
(iii) Number your questions identically as they appear on the question paper and do **NOT write your solutions to different questions** beside each other.
(iv) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED
(a) Mathematical formulae
(b) Scientific calculator (non-programmable, non-graphical)

---

1. (a) Find

(i) $\lim_{x\to 2} \frac{x^3 - 8}{x^2 - x - 2}$ [4]

(ii) $\lim_{x\to 0} \frac{\sin 2x}{\sin 5x}$ [4]

(iii) the value(s) of $x$ for which $f(x) = \frac{4}{x^2-16}$ is discontinuous. [3]

(b) The function $f$ on $\mathbb{R}$ is defined by

$$f(x) = \begin{cases} x^2 - 2x + 3, & x < 3 \\ 4x - 6\ , & x \geq 3 \end{cases}$$

(i) Find

$$\lim_{x\to 3} f(x)$$

[4]

(ii) Determine whether $f(x)$ is continuous at $x = 3$. Give a reason for your answer. [2]

(c) (i) Given that $f(x) = x^2$, determine $f(x + h)$. [2]

(ii) Hence differentiate $f(x) = \frac{1}{x^2}$ from first principles. [6]

Total 25 marks

Please Turn Over

2. (a) Determine $f'(x)$ for each of the following

(i) $f(x) = (2x-1)(x^2+5)^3$ [3]

(ii) $f(x) = \cos(x^2) - \tan(x-5)$ [3]

(b) The curve $y = 2x + \frac{8}{x}$ passes through the point $A(4, 10)$.

Determine

(i) the equation of the normal to the curve at $A$. [5]

(ii) the coordinates of the stationary point(s) on the curve. [4]

(iii) the nature of the stationary point(s). [3]

Total 18 marks

3. (a) The oscillations of a 'baby bouncy cradle' are modelled by the differential equation

$$\frac{dy}{dt} = \frac{150\cos 2t}{y}$$

where $y$ cm is the height of the cradle above its base $t$ seconds after the cradle begins to oscillate. Given that the cradle is 20 cm above its base at time $t = \frac{\pi}{4}$ seconds, show that the particular solution of the differential equation is

$$y^2 = 150\sin 2t + 250$$

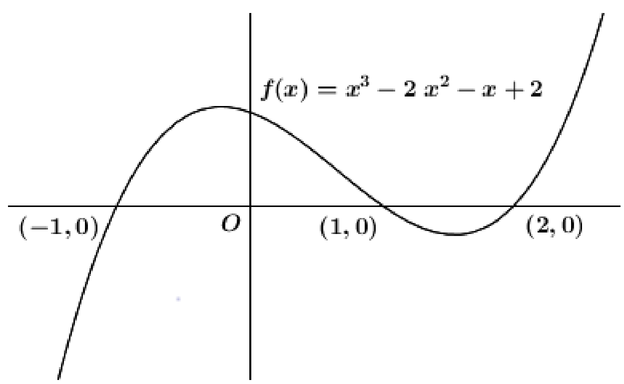
[5]

(b) Using the substitution $u = 2x^3 - 5$, evaluate

$$\int (6x)^2(2x^3-5)^4\,dx$$

[5]

(c)




The diagram above shows a portion of the graph of $f(x) = x^3 - 2x^2 - x + 2$. The graph cuts the $x$-axis at $(-1, 0)$, $(1, 0)$ and $(2, 0)$. Determine the area bounded by $f(x)$, the $x$-axis and the lines $x = 0$ and $x = 2$. [7]

Total 17 marks

**End of Examination**