# HARRISON COLLEGE INTERNAL EXAMINATION, APRIL 2013 CARIBBEAN ADVANCED PROFICIENCY EXAMINATION 

SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 1 - TEST 3
TIME: 1 Hour \& $\mathbf{3 0}$ minutes
This examination paper consists of 3 printed pages.
The paper consists of 3 questions.
The maximum mark for this examination is 60 .

## INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer ALL questions.
3. Number your questions carefully and do NOT write your solutions to different questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact, MUST be written correct to three (3) significant figures.

## EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Electronic calculator (non-programmable, non-graphical)
3. (a) Determine the following limits:
(i) $\lim _{x \rightarrow-2} \frac{x^{2}-x-6}{x+2}$
(ii) $\lim _{x \rightarrow 0} \frac{x^{2}+x}{\sin 4 x}$
(b) Given

$$
f(x)=\left\{\begin{array}{ll}
4 x-p, & x>1 \\
3 & x=1 \\
q x^{2}+1, & x<1
\end{array}\right\}
$$

Find the values of $p$ and $q$ respectively if $f(x)$ is continuous at $x=1$.
(c) Determine from first principles, the derivative with respect to $x$, of $y=\sqrt{x}+2$.
2. (a) Consider $f(x)=\frac{1}{3} x^{3}+2 x^{2}-5 x$. Part of the graph of $f$ is shown below. There is a maximum point at M and a point of inflexion at N .

(i) Find $f^{\prime}(x)$
(ii) Find the $x$-coordinate of $M$.
(iii) Find the $x$-coordinate of N .
(iv) The line $L$ is the tangent to the curve of $f$ at $(3,12)$. Find the equation of $L$ in the form $y=a x+b$.
(b) The percent of concentration, $C(x)$, of a drug in the bloodstream $x$ hours after the drug is administered is given by

$$
C(x)=\frac{4 x}{3 x^{2}+27}
$$

How long after being administered does the concentration start to decrease?
(c) A curve is given parametrically by

$$
x=2 \sin \theta+1 \quad y=1-\cos 2 \theta
$$

Show that $\frac{d y}{d x}=2 \sin \theta$
3. (a) The gradient of a curve is given by $\frac{d y}{d x}=(2 x-3)^{3}$. The point $(2,2)$ lies on the curve. Find the equation of the curve.
(b)
(i) Find $\int_{0}^{2} \sin (4 x+1) d x$
(ii) Using the substitution $u=x-3$, find

$$
\begin{equation*}
\int_{3}^{4} x \sqrt{x-3} d x \tag{5}
\end{equation*}
$$

(c) The shaded region in the diagram below is bounded by $f(x)=\sqrt{x}, x=a$ and the $x$-axis. The shaded region is revolved around the $x$-axis through $360^{\circ}$. The volume of the solid formed is $0.845 \pi$.


Find the value of $a$.
(d) Solve the differential equation $\frac{d y}{d x}=6 x y^{2}$
given that $y=1$ when $x=2$. Give your answer in the form $y=f(x)$.

