HARRISON COLLEGE INTERNAL EXAMINATION, APRIL 2013 CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS UNIT 1 - TEST 3

TIME: 1 Hour & 30 minutes

This examination paper consists of 3 printed pages. The paper consists of 3 questions. The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

- 1. Write your name clearly on each sheet of paper used.
- 2. Answer ALL questions.
- 3. Number your questions carefully and do **NOT** write your solutions to different questions beside one another.
- 4. Unless otherwise stated in the question, any numerical answer that is not <u>exact</u>, **MUST** be written correct to <u>three</u> (3) significant figures.

EXAMINATION MATERIALS ALLOWED

- 1. Mathematical formulae
- 2. Electronic calculator (non-programmable, non-graphical)

1. (a) Determine the following limits:

(i)
$$\lim_{x \to -2} \frac{x^2 - x - 6}{x + 2}$$
 [3]

(ii)
$$\lim_{x \to 0} \frac{x^2 + x}{\sin 4x}$$
 [4]

(b) Given

$$f(x) = \begin{cases} 4x - p, & x > 1\\ 3 & x = 1\\ qx^2 + 1, & x < 1 \end{cases}$$

Find the values of *p* and *q* respectively if f(x) is continuous at x = 1. [4]

(c) Determine from **first principles**, the derivative with respect to *x*, of $y = \sqrt{x} + 2$. [4]

TOTAL 15 marks

2. (a) Consider $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x$. Part of the graph of f is shown below. There is a maximum point at M and a point of inflexion at N.



(i) Find
$$f'(x)$$
 [3]

- (ii) Find the *x*-coordinate of M. [4]
- (iii) Find the *x*-coordinate of N. [3]
- (iv) The line *L* is the tangent to the curve of *f* at (3, 12). Find the equation of *L* in the form y = ax + b. [3]
- (b) The percent of concentration, C(x), of a drug in the bloodstream x hours after the drug is administered is given by

$$C(x) = \frac{4x}{3x^2 + 27}$$

How long after being administered does the concentration start to decrease? [4]

(c) A curve is given parametrically by

$$x = 2sin\theta + 1$$
 $y = 1 - cos2\theta$

Show that
$$\frac{dy}{dx} = 2\sin\theta$$
 [5]

TOTAL 22 marks

3. (a) The gradient of a curve is given by $\frac{dy}{dx} = (2x - 3)^3$. The point (2, 2) lies on the curve. Find the equation of the curve. [4]

(b) (i) Find
$$\int_0^2 \sin(4x+1) dx$$
 [4]

(ii) Using the substitution u = x - 3, find

$$\int_{3}^{4} x\sqrt{x-3} \, dx \tag{5}$$

(c) The shaded region in the diagram below is bounded by $f(x) = \sqrt{x}$, x = a and the *x*-axis. The shaded region is revolved around the *x*-axis through 360°. The volume of the solid formed is 0.845π .



Find the value of *a*.

[5]

(d) Solve the differential equation $\frac{dy}{dx} = 6xy^2$

given that y = 1 when x = 2. Give your answer in the form y = f(x). [5]

TOTAL 23 marks

End of test