UNIT 1 – TEST 3 (2012)

This examination paper consists of 2 pages.
This paper consists of 4 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Do **NOT** do questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

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1. a) Given that $f(x) = \begin{cases} x^2 - 2, & \text{if } x \leq 0 \\ 3x + k, & \text{if } x > 0 \end{cases}$

   i. Evaluate $\lim\limits_{x \to 0^-} f(x)$ [2]

   ii. Find the value for the constant k that will make the function continuous at $x = 0$. [2]

   b) Evaluate the limits

   i. $\lim\limits_{x \to \infty} \left(\dfrac{x^3 - 2x + 3}{6 - 4x^2 - 3x^3} \right)$ [3]

   ii. $\lim\limits_{\theta \to 0} \dfrac{\sin 6\theta}{\sin 5\theta}$ [4]

   c) Evaluate $\lim\limits_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$ [5]

   Hence, find

   i. The gradient of the tangent to $f(x) = x^2$ when $x = 2$ [1]

   ii. The equation of the normal at $x = 2$ [5]

2. a) Determine the derivative of $h(x) = 3x^2 \sin^3(2x)$. [3]

   b) Given that $f(1) = 1$, $g(1) = -2$, $f'(1) = 3$ *and* $g'(1) = -1$.

   Find $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right)$ when $x = 1$ [5]

3. a) A curve has equation $y = x^4 - 32x + 7$

i. Find the x-coordinate of the stationary point of the curve. [4]

ii. State whether the stationary point is a maximum or minimum point. [2]

iii. Hence state the set of values of x for which $x^4 - 32x + 7$ is a decreasing function. [1]

b) A small water balloon was projected vertically upward by a disgruntled calculus student. It reached an elevation of $s = 160t - 16t^2$ feet at the end of t seconds. What is the maximum height the balloon reaches? [5]

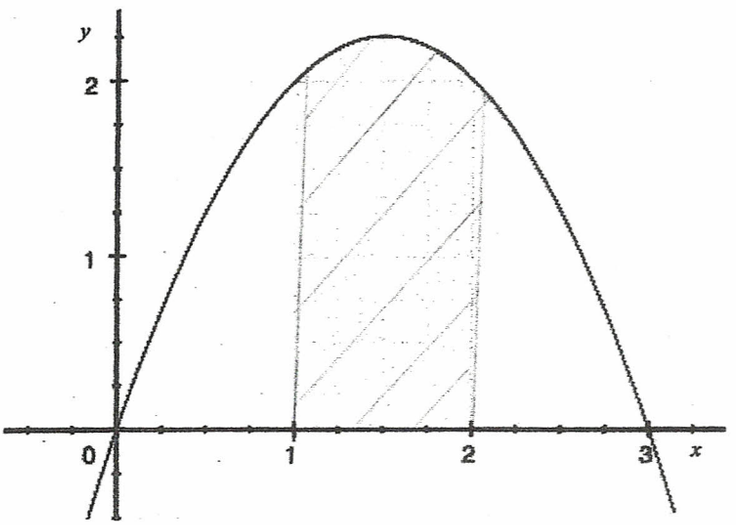
4. a) Evaluate

i. $\int(\sqrt{\theta} - \cos\theta)d\theta$ [2]

ii. $\int(4q^3 - q^2 + q)\ dq$ [3]

b) Using the substitution $u = 3x + 2$, find $\int(3x + 2)^4\ dx$ [4]

c) The curve below has the equation $y = 3x - x^2$.



i. Calculate the area of the shaded region shown for the curve $y = 3x - x^2$ between $x = 1$ and $x = 2$. [4]

ii. Find the volume of the solid generated when the area bounded by, the x axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. State your answer in terms of π. [5]