

**HARRISON COLLEGE INTERNAL EXAMINATION 2011
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION**

SCHOOL BASED ASSESSMENT

**PURE MATHEMATICS
UNIT 1 - TEST 3**

TIME: 1 Hour & 20 minutes

This examination paper consists of 2 printed pages.
The paper consists of 6 questions.
The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Number your questions carefully and do **NOT** write your solutions to different questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

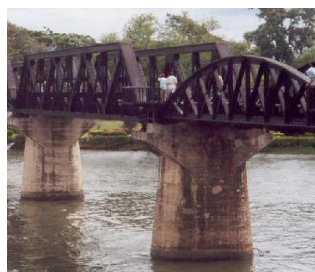
EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
 2. Electronic calculator (non-programmable, non-graphical)
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1. (a) (i) Determine the following limit: $\lim_{x \rightarrow 0} \frac{3 \sin 2x}{x}$ [2]

(ii) Given that $\lim_{x \rightarrow -2} [4f(x)] = 12$
evaluate $\lim_{x \rightarrow -2} [f(x) - 2x]$ [3]

- (b) The position of a portion of the railings of a bridge can be modelled with the following function:



$$f(x) = \left\{ \begin{array}{ll} \frac{9}{4}x + \frac{45}{4} & -5 \leq x \leq -4 \\ \frac{9}{4} & -4 < x \leq -1 \\ -\frac{9}{4}x & -1 < x \leq 0 \\ -\frac{1}{4}(x-3)^2 + \frac{9}{4} & 0 < x \leq 6 \end{array} \right.$$

- (i) Draw a sketch of the graph of $f(x)$. [4]
- (ii) Find $\lim_{x \rightarrow -1^+} f(x)$ [2]
- (iii) Is the function f differentiable at $x = -1$? Explain your answer. [2]

- (c) Determine from **first principles**, the derivative with respect to x , of $y = \sqrt{2x}$. [4]

2. (a) Find the equation of the **normal** to the curve $y = 2x^3 - 4x^2 + 9$ at the point where $x = 3$. [6]

(b) Find $\frac{dy}{dx}$ when:

(i) $y = x\sqrt{x^2 - 1}$ [4]

(ii) $y = \frac{\sin^2 x}{x^2}$ [4]

3. Oil is leaking at a constant rate to form a circular puddle on the floor. The oil is being added to the puddle at the rate of 10 mm^3 per minute causing the puddle to spread out evenly with constant depth 2 mm.

When the radius of the puddle is r mm, the volume is $V \text{ mm}^3$ of oil is given by $V = 2\pi r^2$.

Find the rate of change of the radius of the puddle when the radius is 30 mm. (Give an **exact** answer with units of mm per minute). [4]

4. For the function $f(x) = x - \frac{6}{x} + \frac{9}{x^3}$, $x \neq 0$, determine

(i) the x coordinates of the stationary points, [7]

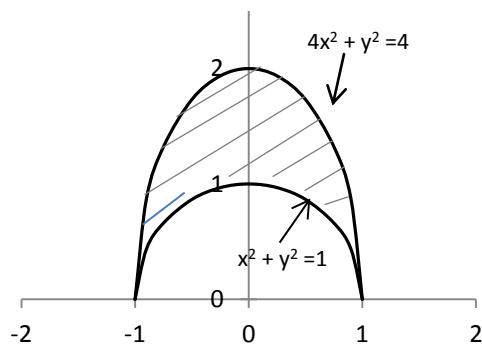
(ii) the nature of each of the stationary points. [3]

5. (i) Find $\int [\tan^2 x + \cos(3x)] dx$ [4]

(ii) Using the substitution $u = x^2 - 1$, find

$$\int \frac{3x}{(x^2-1)^2} dx \quad [4]$$

6.



Determine the exact volume of the solid formed if the area enclosed between the two curves

$$4x^2 + y^2 = 4 \quad \text{and} \quad x^2 + y^2 = 1$$

is rotated by 2π radians about the x -axis. [7]