HC SBA UNIT 1 TEST 2 (2011)

1 (a) $x^2 + y^2 - 3x - 4 = 0$

$$\left(x - \frac{3}{2}\right)^2 + y^2 - 4 - \left(\frac{3}{2}\right)^2 = 0$$

$$\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{25}{4}$$

(i) centre $\left(\frac{3}{2}, 0\right)$

(ii) radius $= \frac{5}{2}$

(iii) on x-axis $y = 0$

$$\therefore \left(x - \frac{3}{2}\right)^2 = \frac{25}{4}$$

$$x - \frac{3}{2} = \pm \frac{5}{2}$$

$x = 4$ and $x = -1$

so coordinates of points on x axis $(4, 0)$ and $(-1, 0)$

on y-axis, $x = 0$

$$\left(-\frac{3}{2}\right)^2 + y^2 = \frac{25}{4}$$

$$y^2 = \frac{25}{4} - \frac{9}{4} = 4$$

$$y = \pm 2$$

so points on y-axis are $(0, 2)$ and $(0, -2)$

1 (b) considering line AB

m_{ab} = gradient = $-\frac{6}{3} = -3$

equation of $L_{AB} \Rightarrow y - 9 = -3(x-2)$

$y = -3x + 15$

considering line CD

$\Rightarrow m_{cd} = \frac{1}{2}$

equation of $L_{CD} \Rightarrow y + 5 = \frac{1}{2}(x-2)$

$y = \frac{1}{2}x - 6$

to find coordinates of D
solve simultaneously

$y = -3x + 15$
$y = \frac{1}{2}x - 6$

$\Rightarrow x = 6 \quad y = -3$

(ii) $5y - 4x = 17$

$y = \frac{4}{5}x + \frac{17}{5} \Rightarrow m = \frac{4}{5}$

gradient of line perpendicular to this line $= -\frac{5}{4}$

equation of line passing through $(6, -3)$

$y + 3 = \frac{-5}{4}(x - 6)$

or $y = \frac{-5x}{4} + \frac{30}{4} - 3$

$y = \frac{-5x}{4} + \frac{18}{4}$

2 (i) $|a| = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$

unit vector in direction of $\underset{\sim}{a}$

$= \frac{1}{5}\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

(ii) $m\begin{pmatrix} 4 \\ -3 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 22 \\ -11 \end{pmatrix}$

$4m + 2 = 22 \Rightarrow m = 5$

(iii) $|b| = \sqrt{4+16} = \sqrt{20}$

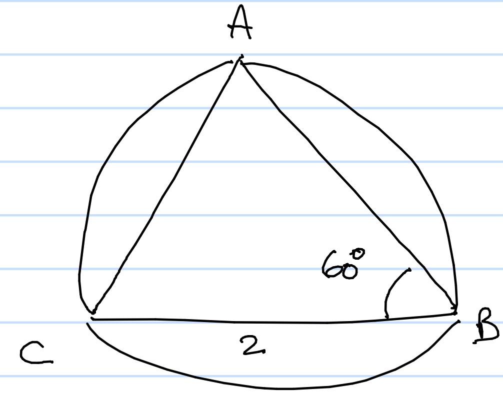
$\underset{\sim}{a} \cdot \underset{\sim}{b} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 8 - 12 = -4$

$\therefore\ 5 \times \sqrt{20}\ \cos\theta = -4$

$\cos\theta = \frac{-4}{5\sqrt{20}}$

$\theta = 124.4^\circ$

3.



(a) $\text{arc } AC = \frac{60}{360} \times 2 \times \pi \times 2 = \frac{2}{3}\pi$

$\therefore$ perimeter $= 3 \times \frac{2\pi}{3} = 2\pi$ cm

3 (b) area of coin =

area of triangle + area of segments

area of segment

$$= \frac{60}{360} \times \pi \times 2^2 - \frac{1}{2} \times 4 \times \sin 60$$

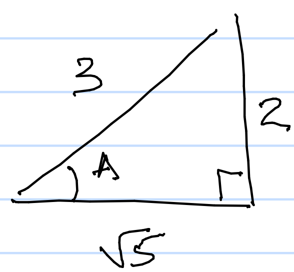
$$= \frac{2}{3}\pi - \frac{2\sqrt{3}}{2} = \frac{2}{3}\pi - \sqrt{3}$$

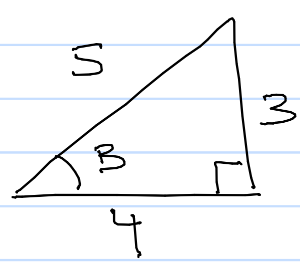
area of coin $= \sqrt{3} + 3\left(\frac{2}{3}\pi - \sqrt{3}\right)$

$$= \sqrt{3} + 2\pi - 3\sqrt{3}$$

$$= 2\pi - 2\sqrt{3} \quad cm^2$$

4 (a)(i)





$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$

$$= \frac{2}{3} \cdot \frac{4}{5} + \frac{3}{5} \cdot \frac{\sqrt{5}}{3}$$

$$= \frac{8}{15} + \frac{3\sqrt{5}}{15} = \frac{8+3\sqrt{5}}{15}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{\sqrt{5}}{3} \cdot \frac{4}{5} - \frac{2}{3} \cdot \frac{3}{5}$$

$$= \frac{4\sqrt{3}}{15} - \frac{6}{15}$$

4(b)(i) $\dfrac{\cot^2\theta}{1+\cot^2\theta}$

$$= \frac{\frac{\cos^2\theta}{\sin^2\theta}}{1 + \frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\cos^2\theta}{\sin^2\theta + \cos^2\theta} = \cos^2\theta$$

b(ii) $\cos^2\theta = 2\cos 2\theta$

$$2\cos 2\theta - \cos^2\theta = 0$$

$$2(2\cos^2\theta - 1) - \cos^2\theta = 0$$

$$4\cos^2\theta - 2 - \cos^2\theta = 0$$

$$3\cos^2\theta - 2 = 0$$

$$\cos^2\theta = \frac{2}{3}$$

$$\cos\theta = \pm\sqrt{\frac{2}{3}}$$

$\theta = 48.1^\circ, -48.1^\circ, 131.9^\circ, -131.9^\circ$

5.

$$\frac{H}{D} = \frac{\frac{v^2}{2g}\sin^2\theta}{\frac{v^2}{g}\sin 2\theta}$$

$$= \frac{1}{2}\frac{\sin^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{1}{4}\tan\theta$$

(b) If $\theta = 30$

$\tan 30 = \frac{\sqrt{3}}{3}$

$\frac{1}{4}\tan\theta = \frac{1}{4}\frac{\sqrt{3}}{3}$

$= \frac{\sqrt{3}}{12}$