HC SBA UNIT 1 TEST 1 (2011)

The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Do **NOT** do questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

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1. a) The equation of a circle is $x^2 + y^2 - 3x - 4 = 0$

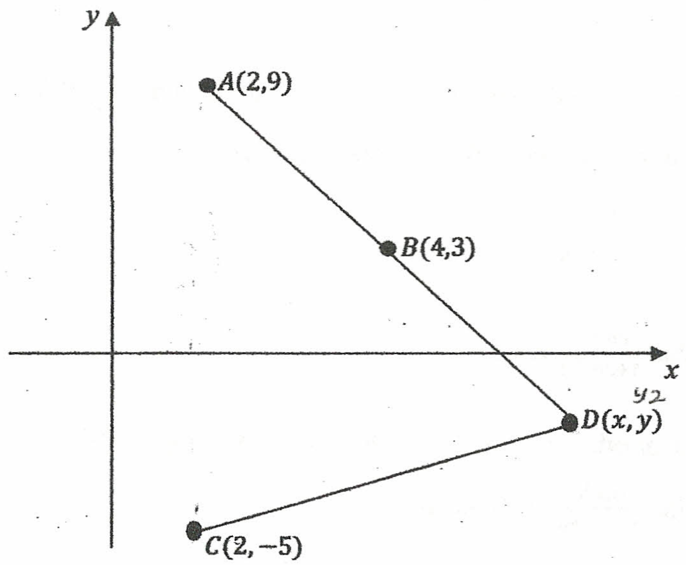
   Find

   i. the coordinates of its centre [3]

   ii. its radius [1]

   iii. the coordinates of the points at which it cuts the axes [7]

   b) Three points have coordinates $A(2,9)$, $B(4,3)$ and $C(2,-5)$ as shown below. The line through C with gradient $\frac{1}{2}$ meets the line AB which is produced to D.



   i. Show that the coordinate of D is $(6,-3)$ [9]

   ii. Hence, find the equation of the line through D which is perpendicular to the line $5y - 4x = 17$. [3]

Total 23 marks

PLEASE TURN OVER

2. Given that $\boldsymbol{a} = \binom{4}{-3}$, $\boldsymbol{b} = \binom{2}{4}$, $\boldsymbol{c} = \binom{22}{-11}$, find

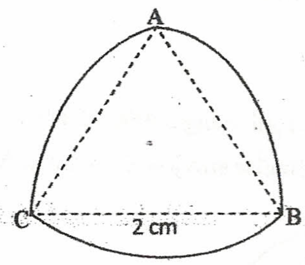
i. A unit vector in the direction of $\boldsymbol{a}$ [3]

ii. The value of the constants m for which $m\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{c}$ [2]

iii. Determine the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. [3]

Total 8 marks

3. A coin is made by starting with an equilateral triangle ABC of side 2 cm. With centre A, an arc of a circle is drawn joining B to C. Similar arcs join C to A and A to B.



Find, leaving your answers as exact values in terms of π

a. the perimeter of the coin [2]

b. the area of one of the coins' faces [7]

Total 9 marks

4. a) A and B are acute angles such that $\sin A = \frac{2}{3}$ and $\tan B = \frac{3}{4}$. Without using a calculator and leaving your answers in surd form, find the value of

i. $\sin(A + B)$ [5]

ii. $\cos(A + B)$ [2]

b) i. Prove that $\frac{\cot^2\theta}{1+\cot^2\theta} \equiv \cos^2\theta$ [3]

ii. Hence, or otherwise, find the solutions in the range $-180° \leq \theta \leq 180°$ of the equation $\frac{\cot^2\theta}{1+\cot^2\theta} = 2\cos 2\theta$. [6]

Total 16 marks

5. Most water fountains have water jets that shoot water into the air to create parabolic arcs. When a stream of water is shot into the air at an angle of θ with the horizontal, then water will travel a horizontal distance of $D = \frac{v^2}{g}\sin 2\theta$ and reach a maximum height of $H = \frac{v^2}{2g}\sin^2\theta$, where g is the acceleration due to gravity.

a) Express the ratio of the maximum height of the water to the horizontal distance it travels, $\frac{H}{D}$, as a function in terms of $\tan\theta$. [3]

b) Show that the ratio of the maximum height of the water to the horizontal distance it travels for an angle of 30° is $\frac{\sqrt{3}}{12}$. [1]