

HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2014
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS

UNIT 1 – TEST 1

1 hour 30 minutes

This examination paper consists of 2 printed pages.

This paper consists of 9 questions.

The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

- (i) Write your name clearly on each sheet of paper used
- (ii) Answer **ALL** questions
- (iii) Number your questions identically as they appear on the question paper and do **NOT** write your solutions to different questions beside each other
- (iv) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

- (a) Mathematical formulae
- (b) Scientific calculator (non-programmable, non-graphical)

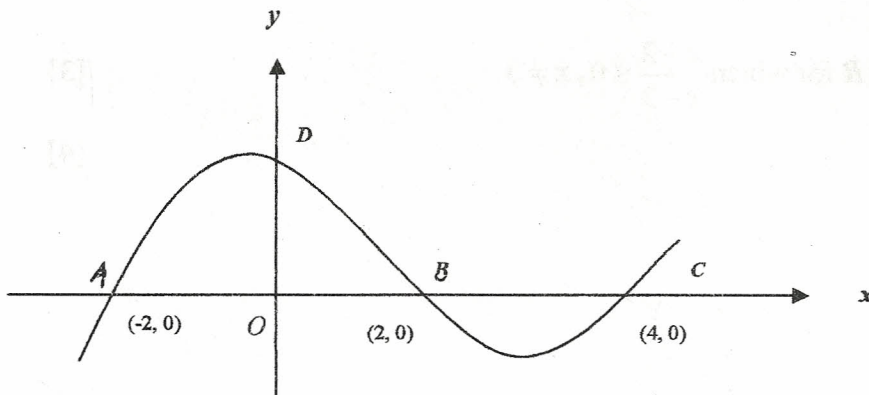
1) Given that p and q are propositions, use the algebra of propositions to show that $(p \wedge q) \vee (\sim p \wedge q) \equiv q$ [3]

2) (i) Evaluate $\sum_{r=1}^{500} (3r + 2)$. [4]

(ii) Given that $\sum_{r=1}^n (ar + b) = n^2$, find the constants a and b . [4]

3) (a) The sketch below, **not drawn to scale**, shows part of the graph of

$y = x^3 + px^2 + qx + r$, where p , q and r are constants.



Please Turn Over

The points A , B and C have coordinates $(-2, 0)$, $(2, 0)$ and $(4, 0)$ respectively. The curve crosses the y -axis at D .

- (i) Evaluate p , q and r . [6]
(ii) Determine the coordinates of D . [1]

(b) Given the polynomial $f(x) = x^3 - 2x^2 - x + 2$, solve $f(x) = 0$ for $x \in \mathbf{R}$. [6]

4) Prove by mathematical induction that $\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1} \forall n \in \mathbf{Z}^+$. [6]

5) (a) By using the substitution $u = 2^x$, solve the equation $4^x - 10(2^x) + 16 = 0$. [4]

(b) Solve for x the equation $e^{2x} + 4e^{-2x} = 4$, giving your answer in terms of \ln . [4]

6) The population of a town at the beginning of the year 2000 was 2400. The population increased so that, after a period of n years, the new population was $2400(1.06)^n$. Calculate estimates of

(i) the population at the beginning of 2010 [2]

(ii) the year in which the population is expected to first reached 7000. [4]

7) The function f is defined by $f: x \rightarrow \ln(x+1); x \in \mathbf{R}, x > -1$.

(i) Sketch the graph of f , showing clearly any intersection with the axes. [2]

(ii) Determine an expression for the inverse function, $f^{-1}(x)$. [3]

(iii) State the domain, and the range of $f^{-1}(x)$. [2]

The function g is defined by $g: x \rightarrow x - 1, x \in \mathbf{R}$.

(iv) Determine $fg(x)$. [2]

8) Find the range of values of $x \in \mathbf{R}$ for which $\frac{x-2}{x-3} \leq 0, x \neq 3$ [3]

9) Solve for $x \in \mathbf{R}, x = |3x| - 2$ [4]

End of Test