SBA UNIT 1 - TEST 1 (2013)

1)

p	$\sim p$	$p \vee \sim p$
1	0	1
0	1	1

statement is a tautology

2)
$$\frac{5\sqrt{2}+1}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{10\sqrt{2}+2+10+\sqrt{2}}{4-2}$$

$$= \frac{12+11\sqrt{2}}{2} = 6+\frac{11}{2}\sqrt{2}$$

3) Let $P(n)$ be the statement that

$$\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2) \quad \forall n \in Z^+$$

now for $n=1$

$$\sum_{r=1}^{1} r(r+1) = \frac{1}{3}(1)(1+1)(1+2)$$

$$1(1+1) = \frac{1}{3}(1)(2)(3)$$

$$2 = 2$$

so $P(1)$ is true

3 (continued)

Let $n = k$ assume $P(k)$ is true

i.e. $\sum_{r=1}^{k} r(r+1) = \frac{1}{3}k(k+1)(k+2)$

now for $n = k+1$

$$\sum_{r=1}^{k+1} r(r+1) = \sum_{r=1}^{k} r(r+1) + (k+1)(k+2)$$

$$= \frac{1}{3}k(k+1)(k+2) + (k+1)(k+2)$$

$$= (k+1)(k+2)\left(\tfrac{1}{3}k + 1\right)$$

$$= \frac{1}{3}(k+1)(k+2)(k+3)$$

So $P(k+1)$ is true when $P(k)$ is true.

So since $P(1)$ is true and $P(k+1)$ is true whenever $P(k)$ is true, $P(1)$ is true, $P(2)$ is true, $P(3)$ is true etc

$\therefore P(n)$ is true $\forall n \in \mathbb{Z}^+$.

4) If $(x+2)$ is a factor $\Rightarrow x = -2$ is a root

$\therefore f(-2) = 0$; $2(-2)^3 + 5(-2)^2 + (-2)a - 6 = 0$

$-16 + 20 - 2a - 6 = 0$

$a = -1$

4 a(ii)

$$
\begin{array}{r|l}
 & 2x^2 + x - 3 \\
\hline
x+2 & 2x^3 + 5x^2 - x - 6 \\
 & \underline{2x^3 + 4x^2} \\
 & \quad x^2 - x \\
 & \quad \underline{x^2 + 2x} \\
 & \qquad -3x - 6 \\
 & \qquad \underline{-3x - 6} \\
 & \qquad = \quad =
\end{array}
$$

$$2x^2 + x - 3 = (2x+3)(x-1)$$

So other ROOTS of $f(x) = 0$ are

$x = -\frac{3}{2}$ and $x = 1$

(b) $2x^3 - 54 = 2(x^3 - 27)$

$= 2(x^3 - 3^3)$ so $(x-3)$ is a factor

$= 2(x-3)(x^2 + 3x + 9)$

5. $2e^{2x} + e^{x} - 10 = 0$

$(2e^{x} + 5)(e^{x} - 2) = 0$

$e^{x} = -\frac{5}{2} \Rightarrow x = \ln\left(-\frac{5}{2}\right)$ NO SOLUTION

$e^{x} = 2 \Rightarrow x = \ln 2$

Ans: $x = \ln 2$

6. $h(t) = 2 + \log_{10}(t+3)$

at 6:37 a.m., $t = 97$ minutes

$h(t) = 2 + \log_{10}(97+3)$

$= 2 + \log_{10}(100)$

$= 2 + 2 = 4$ joules

6(ii) $2 + \log_{10}(t+3) = 5$

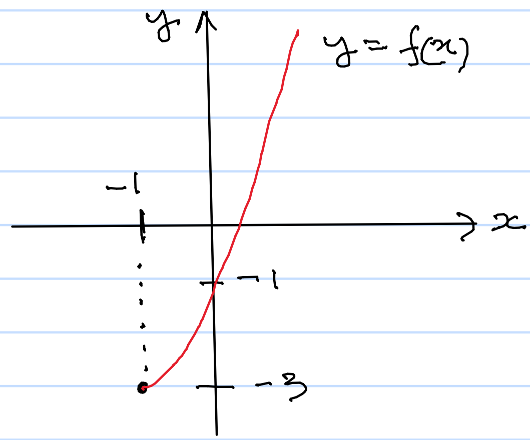
$\log_{10}(t+3) = 3$

$t + 3 = 10^3 = 1000$

$t = 1000 - 3 = 997$ minutes

7 (i) $f(x) = 2(x+1)^2 - 3$

(ii)



(iii) range of f: $y \geq 3$

(iv) f is injective. f is 1-1, passes the horizontal line test.

f is surjective. f is onto all elements of the co domain are mápped onto.

f is bijective. f is both injective and surjective.

f has an inverse. f is injective.

8. Let $X = \frac{1}{x} \Rightarrow x = \frac{1}{X}$

substituting

$$4\frac{1}{X^3} - 2\frac{1}{X^2} + 5\frac{1}{X} + 6 = 0$$

multiply by X^3

$$4 - 2X + 5X^2 + 6X^3 = 0$$

this equation has roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$

sum of the roots

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -\frac{5}{6}$$

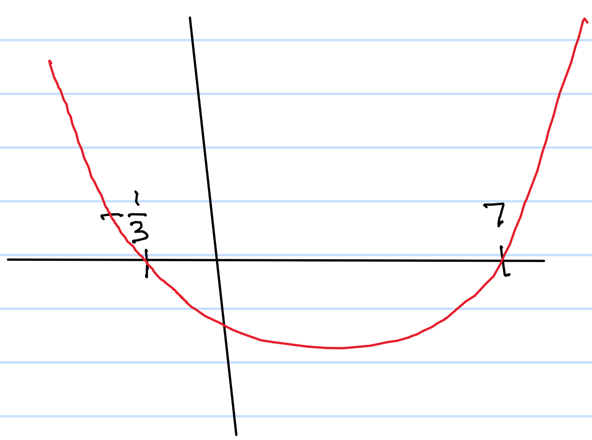
9. $|3-2x| > |x+4|$

square both sides

$$9 - 12x + 4x^2 > x^2 + 8x + 16$$

$$3x^2 - 20x - 7 > 0$$

$$(3x+1)(x-7) > 0$$



So solution $\{x < -\frac{1}{3}\} \cup \{x > 7\}$