

HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2013
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 1 – TEST 1
1 hour 30 minutes

This examination paper consists of 2 printed pages.

This paper consists of 9 questions.

The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

- (i) Write your name clearly on each sheet of paper used
- (ii) Answer ALL questions
- (iii) Number your questions identically as they appear on the question paper and do **NOT** write your solutions to different questions beside each other
- (iv) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

- (i) Mathematical formulae
- (ii) Scientific calculator (non-programmable, non-graphical)

1. Use a truth table to determine whether the statement $\sim p \vee p$ is a tautology or a contradiction.

[3]

Total 3 marks

2. Express $\frac{5\sqrt{2}+1}{2-\sqrt{2}}$ in the form $p + q\sqrt{2}$ where p and q are real numbers.

[5]

Total 5 marks

3. Prove by mathematical induction that $\sum_{r=1}^n r(r+1) = \frac{1}{3}n(n+1)(n+2) \forall n \in \mathbb{Z}^+$.

[8]

Total 8 marks

4. The function $f(x) = 2x^3 + 5x^2 + ax - 6$, where a is a constant, has $(x + 2)$ as a factor.

(a) (i) Find the value of a .

[3]

(ii) When a has this value, find all the other roots of the equation $f(x) = 0$.

[5]

(b) Factorise completely $2x^3 - 54$

[3]

Total 11 marks

5. Solve, giving the exact value(s) of $x \in \mathbb{R}$, the equation $2e^{2x} + e^x - 10 = 0$.

[5]

Total 5 marks

P.T.O

6. The heat supplied by a solar panel is modelled by the equation $h(t) = 2 + \log_{10}(t + 3)$, where $h(t)$ is the amount of heat, in Joules, supplied at time t minutes after 5.00 a.m.

Calculate

- (i) the amount of heat supplied by the panel at 6.37 a.m. [2]
(ii) the length of time it takes, in minutes, for the panel to supply 5 joules of heat. [3]

Total 5 marks

7. The function f is defined by $f: x \rightarrow 2x^2 + 4x - 1, x \geq -1, f(x) \geq -3, (x, f(x)) \in \mathcal{R}$.

- (i) Express f in the form $a(x + h)^2 + k$, where a, h and k are constants [4]
(ii) Sketch the graph of f [2]
(iii) State the range of f [1]
(iv) Giving a clear and concise reason to support your answer, state whether f
(a) is injective [2]
(b) is surjective [2]
(c) is bijective. [2]
(d) has an inverse. [2]

Total 15 marks

8. Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ if α, β and γ are the roots of the cubic equation

$$4x^3 - 2x^2 + 5x + 6 = 0 \quad [4]$$

Total 4 marks

9. Find the range of values of x for which $|3 - 2x| > |x + 4|$. [4]

Total 4 marks

End of Test