HARRISON COLLEGE

END OF YEAR EXAMINATION

FOURTH YEAR MATHEMATICS

PAPER 02

DURATION: 2 Hours

INSTRUCTIONS TO CANDIDATES

1) This question paper consists of **SIX** printed pages and 10 questions.
2) Write your name clearly on **EACH** sheet of paper used.
3) All questions are to be attempted.
4) All working must be clearly shown.
5) Number your responses carefully and identically (including any associated parts) as they appear on the question paper.

   DO NOT WRITE ANY responses beside each other.
6) Calculators are allowed.
7) If a numerical answer cannot be given **exactly**, and the accuracy required is not specified in the question, then in the case of an angle, it must be given correct to one (1) decimal, and in others cases it must be given correct to three (3) significant figures.
8) The maximum mark for this examination is 85.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

CSEC Mathematics
LIST OF FORMULAE

Volume of Prism $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of Cylinder $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference $C = 2\pi r$ where r is the radius of the circle.

Arc length $S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.

Area of a circle $A = \pi r^2$ where r is the radius of the circle.

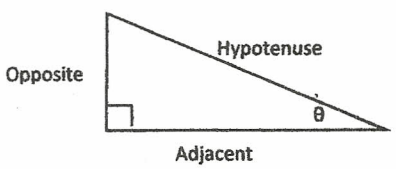
Area of a sector $A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.

Area of Trapezium $A = \frac{1}{2}(a+b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios $\sin\theta = \frac{opposite\ \ side}{hypotenuse}$

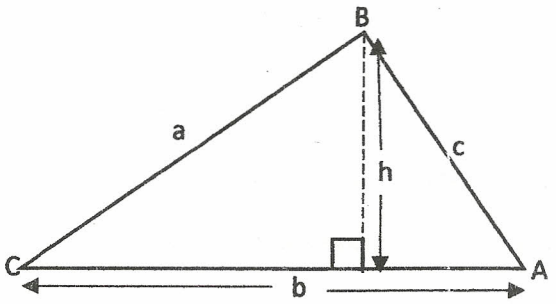
$\cos\theta = \frac{adjacent\ \ side}{hypotenuse}$



$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Area of a triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height



Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

1. A survey was conducted among 40 households. The results were:
   30 owned laptops (L)

   28 owned desktops (D)

   $3x$ owned both laptops and desktops

   x owned neither

i) Draw a Venn diagram to represent the information given above. [4]

ii) Write an expression, in x, to represent the TOTAL number of households in the survey. [2]

iii) Calculate the value of x. [2]

iv) Determine the number of households which own laptops only. [2]

Total 10 marks

2. Factorise the following completely:

a) $12ab - 16ac$ [1]

b) $4x^3y - 9y^3x$ [3]

c) $2x^2 + 4xy - x - 2y$ [3]

d) $6x^2 - 17x + 12$ [3]

Total 10 marks

3. Solve the following equations:

a) $5x^2 + 2x = 0$ [3]

b) $4x^2 = 7x - 3$ [3]

c) $(x + 1)(x - 10) + 30 = 0$ [4]

d) $3x^2 - 4x - 13 = 0$ correct to 2 decimal places [4]

Total 14 marks

4. Given that $f(x) = 6x^2 - 10x - 3$

i) Write $f(x)$ in the form $f(x) = a(x + h)^2 + k, where\ a, h\ and\ k \in \mathbb{R}$. [3]

ii) State the equation of the axis of symmetry. [1]

iii) State the coordinates of the minimum point. [1]

Total 5 marks

5. The position vectors of the points R, S *and* T relative to the origin are

$$\overrightarrow{OR} = \begin{pmatrix} 7 \\ 8 \end{pmatrix}, \overrightarrow{OS} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \text{ and } \overrightarrow{OT} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$$

i) Express in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vectors

a) $\overrightarrow{RT}$ [2]

b) $\overrightarrow{SR}$ [2]

ii) The point F is such that $RF = FT$. Use a vector method to determine the coordinates of the point F.

[5]

Total 9 marks

6. Solve the pair of simultaneous equations

$$y + 4x = 27$$
$$xy + x = 40$$

Total 6 marks

7. Make r the subject of the following formulae:

$$a)\ p = 4r + 6t$$

[2]

$$b)\ \sqrt{pr} = \frac{3g}{h+2}$$

[2]

$$c)\ R = \frac{2r-1}{3r+2}$$

[4]

Total 8 marks

8. The functions f *and* g are defined by

$$f(x) = 6x + 8\,; g(x) = \frac{x-2}{3}$$

i) Calculate the value of $g\left(\frac{1}{2}\right)$. [2]

ii) Write an expression for $gf(x)$ in its simplest form. [2]

iii) Find the inverse function $f^{-1}(x)$. [2]

Total 6 marks

9. i) Sketch a diagram to represent the information given below. Clearly show all measurements.

P, Q and R are three buildings.

Q is 155 m due west of P.

The bearing of R from Q is 190^0.

RQ is 85 m. [5]

ii) Calculate , to one decimal place, the distance RP. [3]

iii) Calculate the sizes of the angles QRP and QPR. [3]

iv) Calculate, to the nearest degree, the bearing of R from P. [2]

Total 13 marks

10. A, B and C are matrices such that:

$$A = (3 \quad -4), B = \begin{pmatrix} 2 & x \\ y & -7 \end{pmatrix} \text{ and } C = (-10 \quad 1)$$

Given that $AB = C$, calculate the values of x *and* y.

Total marks [4]

END OF TEST