FOURTH YEAR MATHEMATICS PAPER 2008 – 2009

Time: 2 hours

INSTRUCTIONS

Answer ALL questions

Write your name clearly on each sheet of paper used.

Number your answers carefully and do NOT do questions beside one another.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Omissions of essential working will result in loss of marks.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to 2 decimal places.

Formulae and graph paper are provided. Mathematical tables or electronic calculators may be used to evaluate explicit numerical expressions.

The number of marks available is shown in the [ ] at the end of each question.

LIST OF FORMULAE

Volume of a prism $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of a right pyramid $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference $C = 2\pi r$ where r is the radius of the circle.

Area of a circle $A = \pi r^2$ where r is the radius of the circle.

Area of trapezium $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations If $ax^2 + bx + c = 0$,

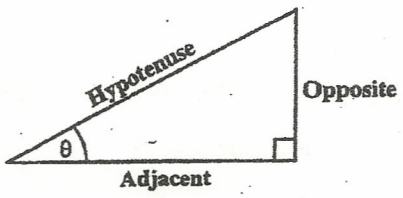
then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

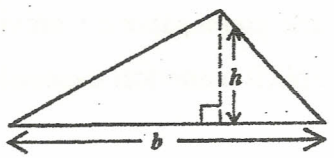
Trigonometric ratios

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



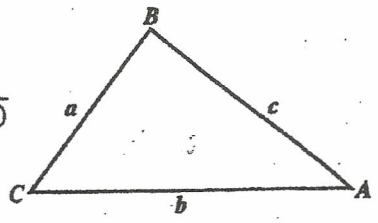


Area of triangle Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height

Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a + b + c}{2}$



Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

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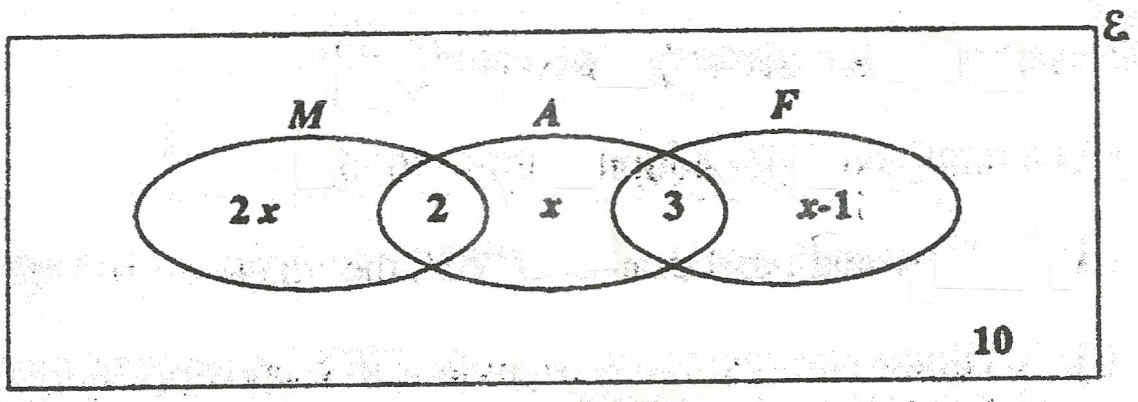


1. The Venn diagram above shows the number of students doing mathematics (M), accounts (A) and French (F) in a class of 50.

   i. Write down an expression in terms of x for the number of students who do accounts. **[2]**

   ii. Write down an equation in terms of x which shows the information in the Venn diagram. **[2]**

   iii. Determine the number of students who do maths only. **[2]**

   iv. Determine the number of students who do French. **[3]**

2.

   a. Factorise completely:

   i. $x^2 + 12x + 36$ **[2]**

   ii. $6rx - 6ry + 5sx - 5sy$ **[2]**

   iii. $81x^2 - 1$ **[2]**

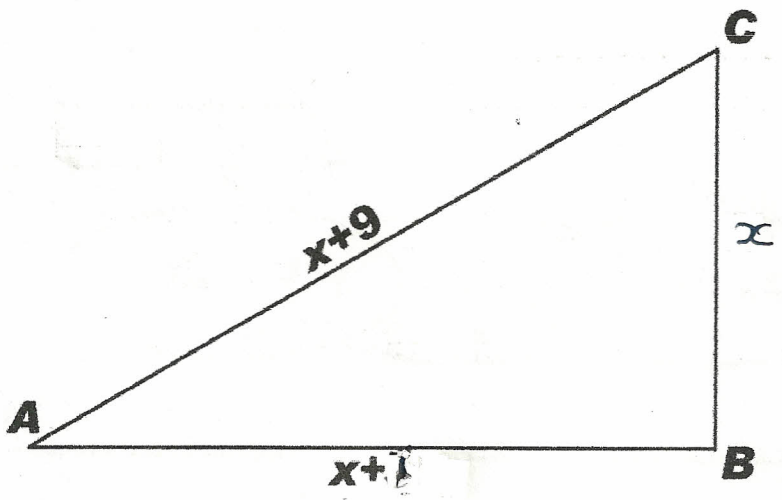
   b. Simplify the following expressions:

   i. $7x^{-2}y^{-3} \times 2x^3y^{-1} \times 3x^4y^5$ **[2]**

   ii. $x^{-\frac{3}{4}}y^{\frac{1}{2}} \times x^{-\frac{3}{4}}y^{-1}$ **[2]**

   iii. $9x^5y^{-3} \div 3x^{-2}y$

   [2]



3.

a. The three sides of a right-angled triangle are as shown in the diagram above.

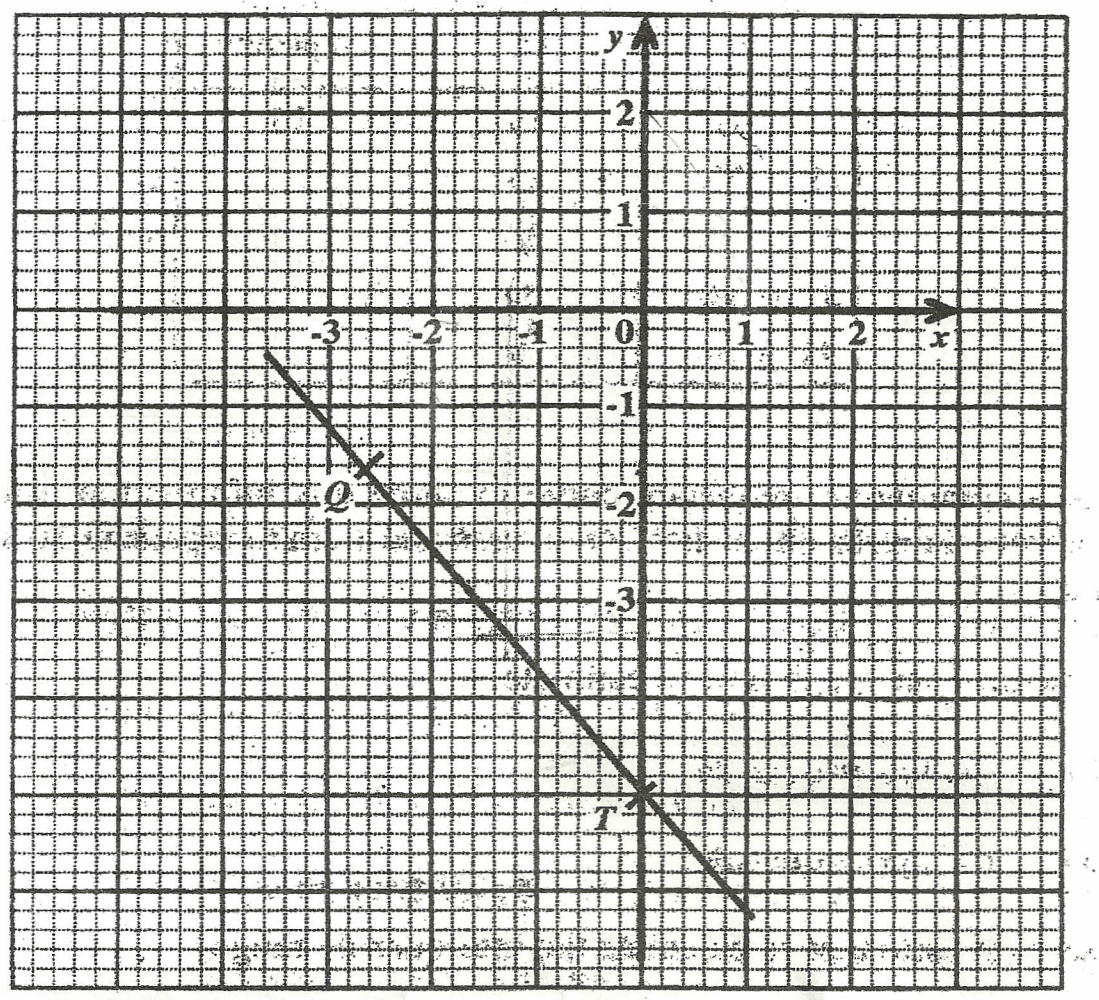
1. Form a quadratic equation in x and [3]
2. solve to determine the length of the three sides [3]

b. Given that the position vectors $\overrightarrow{OA} = \begin{pmatrix} -6 \\ 5 \end{pmatrix}$, and $\overrightarrow{OC} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ are the adjacent sides of a parallelogram OABC. Determine:

i. The resultant $\overrightarrow{OB}$ of the two vectors. [2]

ii. The magnitude $|\overrightarrow{OB}|$ of $\overrightarrow{OB}$ [4]

iii. The acute angle between $\overrightarrow{OB}$ and the x-axis. [4]



4.

a. The graph above shows a straight line QT, intersecting the Y-axis at T.

i. State the co-ordinates of T **[2]**

ii. Calculate the gradient of QT. **[3]**

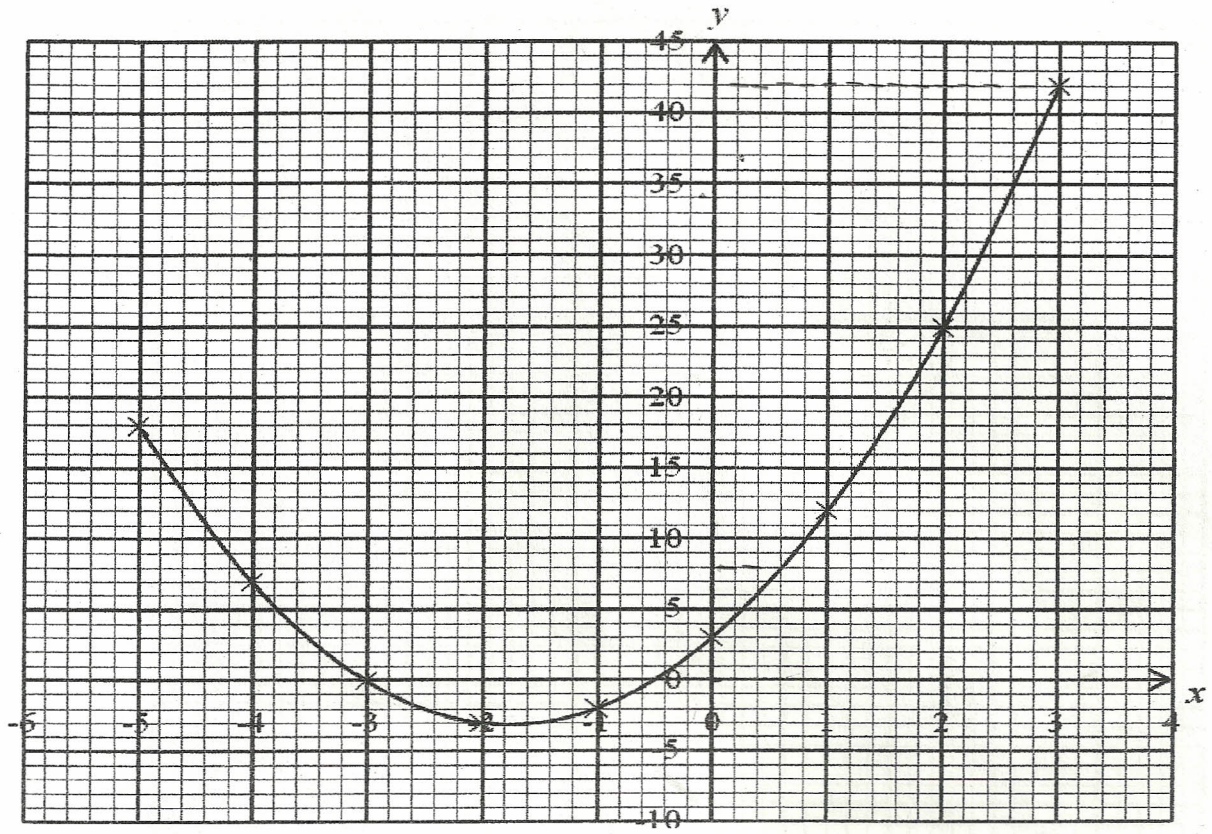
iii. determine the equation of QT **[3]**

iv. What are the co-ordinates at M the mid-point of QT? **[3]**

b. Write the equation $3x^2 + 8x - 25$ in the form $a(x+h)^2 + k$, where a, h and k are real numbers. **[5]**

i. Hence state the minimum value of $3x^2 + 8x - 25$. **[1]**

ii. State its corresponding x-value. **[3]**



5.

a. Using the graph of $f(x) = 2x^2 + 7x + 3$ shown above:

i. find the values of x for which $f(x) = 0$ **[2]**

ii. state the interval of the domain for which: **[2]**

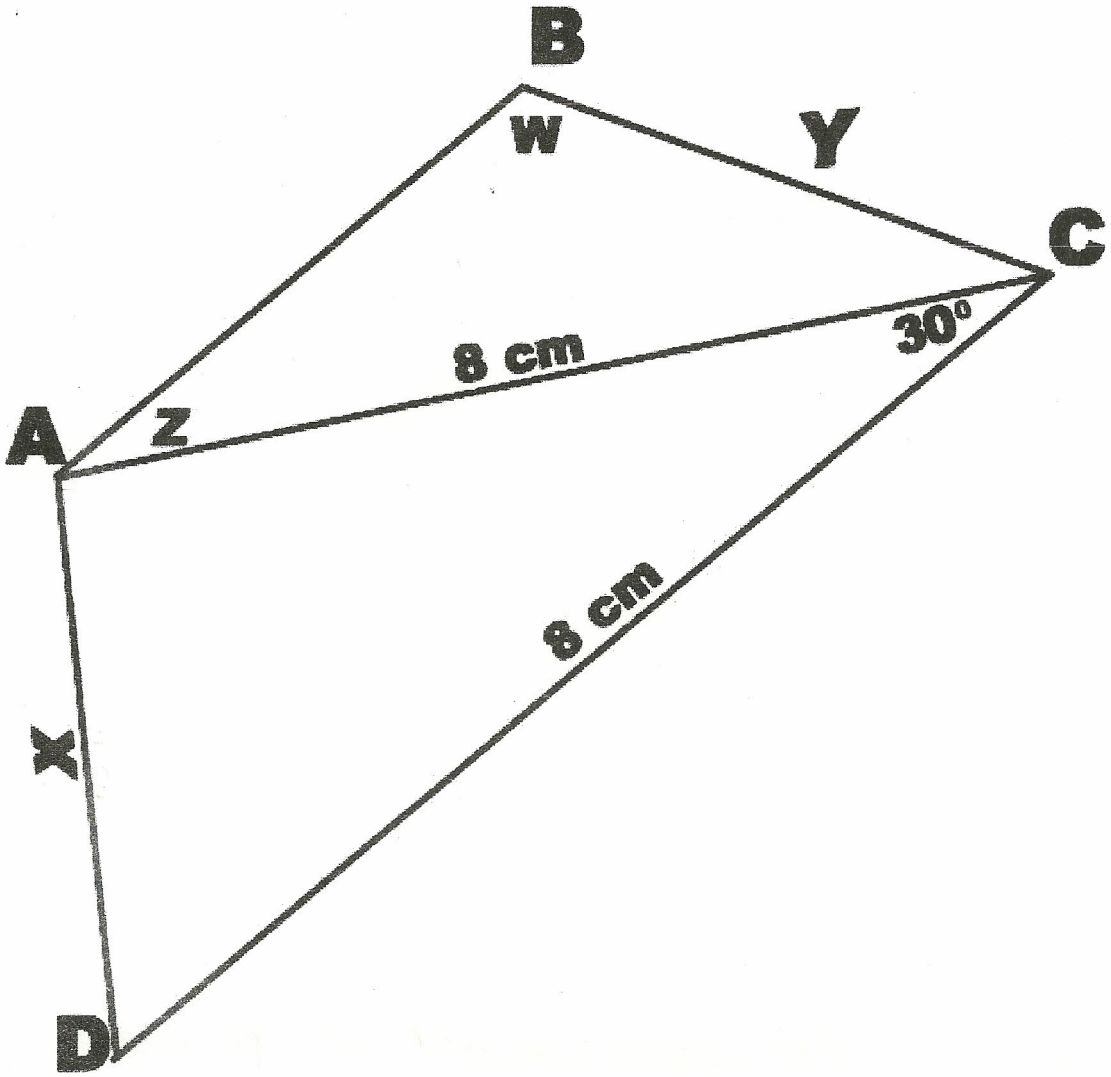
(a) $f(x) > 0$ **[2]**

(b) $f(x) < 7$ **[2]**

b. Given the function $f(x) = \dfrac{3x+1}{x-1}$ and $g(x) = 2x - 3$ evaluate:

i. $fg(3)$ **[3]**

ii. $f^{-1}(2)$



6.

In the figure ABCD above, **not drawn to scale,** AB is parallel to DC. ΔABC and ΔACD are both isosceles. AC = CD = 8 cm, and ∠ACD = 30°.

a. Find area of Δ ACD **[4]**

b. Find x **[4]**

c. Find ∠Z, hence find w **[2]**

d. Find y **[4]**