HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2019

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 2 – TEST 2 PREVIEW

1 hour 20 minutes

1. An arithmetic series has first term a and common difference d.

   The sum of the first 29 terms of the series is 1102.

   a) Show that $a + 14d = 38$. [3]

   b) The sum of the second term and the seventh term is 13. Find the value of a and the value of d. [4]

   $[a = -4, d = 3]$

2. Mr. Marshall will be paid a salary of $35 000 in the year 2015. Mr. Marshall's contract promises a 4% increase in salary every year, the first increase being given in 2016.

   a) Find, to the nearest $100, Mr. Marshall's salary in the year 2018. [2]

   b) Mr. Marshall will receive a salary each year from 2015 until he retires at the end of 2034. Find, to the nearest $1000, the total amount of salary he will receive in the period from 2015 until he retires at the end of 2034. [4]

   [a) 39 400, b) 1042 000]

3. a) Show that the equation $2x^3 + 6x - 1 = 0$ has a root α, between 0 and 1. [3]

   b) A first approximation to α is 0.5. Find a better approximation to the root, α, to one decimal place. [3]

   [b) 0.2]

4. Find the coefficient of x^6 in the expansion of $(1-3x)(1+2x)^9$ as a series of ascending powers of x. [6]

   [-6720]

5. a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, |x| < \frac{8}{3},$$

in ascending powers of x, up to and including the term in x^3, giving each term as a simplified fraction.

[5]

b) Use your expansion, with a suitable value of x, to obtain an approximation to $\sqrt[3]{7.7}$. Give your answer to 7 decimal places. [2]

[a) 2-$\frac{1}{4}x - \frac{1}{32}x^2 - \frac{5}{768}x^3 - \cdots, b)$ 1.9746810]

6. a) Obtain the first four non-zero terms in the Maclaurin series expansion of $f(x) = \cos 2x$. [4]

   b) Hence, using the double angle formula for $\cos 2x$, find the first four non-zero terms in the expansion of cos^2x. [3]

   c) Use the result from b) to show that

$$\lim_{x\to 0}\frac{cos^2x + x^2 - 1}{x^4} = \frac{1}{3}$$

[2]

[a) $1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \cdots$, b) $1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6 + \cdots$,]

7. a) Verify the identity

$$\frac{1}{k} - \frac{1}{k+1} \equiv \frac{1}{k(k+1)}$$

[2]

b) Hence, using the method of summation by differences, show that

$$\sum_{r=1}^{n}\left(\frac{1}{k(k+1)}\right) = 1 - \frac{1}{n+1}$$

[4]

c) Deduce the value of

$$\sum_{r=1}^{\infty}\left(\frac{1}{k(k+1)}\right)$$

[2]

[c) 1]

8. Prove by mathematical induction that [8]

$$\sum_{r=1}^{n} r \times r! = (n+1)! - 1$$