

**UNIT 1 – TEST 2, PREVIEW 2017**

**Time: 1 Hour & 20 minutes**

(Please note that changes have been made to questions 4 and 6)

1. Solve, for  $0 \leq \theta \leq 2\pi$ , the equation  $6 \sin^2 \theta - \cos \theta = 4$  [7]

Total 7 marks

2. Prove that  $\tan A \equiv \frac{1 - \cos 2A + \sin 2A}{1 + \cos 2A + \sin 2A}$  [5]

Total 5 marks

3. Given that  $\sin A = \frac{12}{13}$  and  $\sin B = \frac{4}{5}$ , where  $A$  and  $B$  are both acute angles, find the exact value of

$\sin(A - B)$ . [6]

Total 6 marks

4. Express  $\sin x + \sqrt{3} \cos x$  in the form  $R \sin(x + \alpha)$ , where  $R > 0$  and  $0^\circ \leq \alpha \leq 90^\circ$ , giving the exact value of  $R$  and the values of  $\alpha$  correct to 1 decimal place.

Hence solve the equation  $\sin x + \sqrt{3} \cos x + 1 = 0$  for  $0^\circ \leq x \leq 360^\circ$ . [6]

Total 6 marks

5. i) The centre of a circle is  $(-2, 3)$  and a point on the circumference is  $(-5, -1)$ . Find the equation of the circle. [4]

ii) Determine the equation of the tangent to the circle at the point  $(-5, -1)$ . [4]

iii) Find the points of intersection of the circle in part i) with the circle with equation  $x^2 + y^2 + 6x - 7y - 10 = 0$  [6]

Total 14 marks

6. i) Determine the vector equation of the line joining the points  $(2, 3, -2)$  and  $(0, 7, -1)$ .

[3]

- ii) Find the point of intersection of the straight line with equation  $\mathbf{r} = \begin{pmatrix} -6 \\ -3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix}$  intersects

the line from part i).

[5]

- iii) Determine the angle between the two lines.

[3]

Total 11 marks

7. With respect to an origin  $O$ , the points  $A$  and  $B$  have position vectors  $\mathbf{a} = -4\mathbf{i} + 4\mathbf{j} - \mathbf{k}$  and  $\mathbf{b} = 5\mathbf{i} - 2\mathbf{j} + 11\mathbf{k}$  respectively.

- i) Find the distance between the points  $A$  and  $B$ .

[3]

- ii) Find the equation of the plane  $\pi$  passing through the point  $2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$  and perpendicular to  $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ .

[3]

- iii) Find the angle between the line  $AB$  and the plane  $\pi$ .

[5]

Total 11 marks

END OF TEST

## Answers

1.  $2.30, 3.98, \frac{\pi}{3}, \frac{5\pi}{3}$

2. Proof

3.  $\frac{16}{65}$

4.  $2\sin(x + 60^\circ), 150^\circ, 270^\circ$

5. i)  $(x + 2)^2 + (y - 3)^2 = 25$

ii)  $y = -\frac{3}{4}x - \frac{19}{4}$  or  $4y = -3x - 19$       iii)  $(2, 6)(-2, -2)$

6. i)  $\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -4 \\ -1 \end{pmatrix}$       ii)  $(4, -1, -3)$       iii)  $71.4^\circ$

7. i)  $\sqrt{261}$  or 16.2 units      ii)  $r \cdot \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} = -7$       iii)  $13.1^\circ$