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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

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06 JUNE 2017 (a.m.)

FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

TEST CODE

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SUBJECT PURE MATHEMATICS – UNIT 2 – Paper 032

PROFICIENCY ADVANCED

REGISTRATION NUMBER

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SCHOOL/CENTRE NUMBER

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NAME OF SCHOOL/CENTRE

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CANDIDATE’S FULL NAME (FIRST, MIDDLE, LAST)

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DATE OF BIRTH

D	D	M	M	Y	Y	Y	Y
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SIGNATURE _____

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FORM TP 2017301



TEST CODE 02234032

MAY/JUNE 2017

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2 – Paper 032

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of ONE question.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – Revised 2012

Mathematical instruments

Silent, non-programmable, electronic calculator

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SECTION A

Module 1

Answer this question.

1. (a) Use the trapezium rule with 5 ordinates (4 divisions) to estimate $\int_0^1 \sqrt{1+x^3} dx$.

$$f(x) = \sqrt{1+x^3}$$

$$f(0) = 1$$

$$f(0.25) = 1.00778$$

$$f(0.5) = 1.06066$$

$$f(0.75) = 1.19242$$

$$f(1) = 1.41421$$

$$\begin{aligned} \text{Area} &= \frac{1-0}{2(4)} \left[(1 + 1.41421) + 2(1.00778 + 1.06066 + 1.19242) \right] \\ &= 1.08289 \end{aligned}$$

[4 marks]

GO ON TO THE NEXT PAGE

(b) A complex number is given as $z = 4 - 4i$. Show that $z^8 = 32^4$.

$$|z| = \sqrt{4^2 + (-4)^2} = 4\sqrt{2}$$

$$\arg z = \pi - \tan^{-1}(1) = \frac{3\pi}{4}$$

$$z = 4\sqrt{2} \left(\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right)$$

$$z^8 = (4\sqrt{2})^8 \left(\cos(6\pi) + i \sin(6\pi) \right)$$

$$= 32^4 (1)$$

$$= 32^4$$

[6 marks]

GO ON TO THE NEXT PAGE

(c) Show that the equation of the tangent to the curve

$$\tan(x+y) = \sin(xy) \text{ at the point } (\sqrt{\pi}, -\sqrt{\pi}) \text{ is the line } y = \frac{\sqrt{\pi}-1}{1+\sqrt{\pi}}x - \frac{2\pi}{1+\sqrt{\pi}}.$$

$$\begin{aligned} \tan(x+y) &= \sin(xy) \\ \sec^2(x+y) \left(1 + \frac{dy}{dx}\right) &= \cos(xy) \left(y + x \frac{dy}{dx}\right) \end{aligned}$$

$$\sec^2(x+y) + \sec^2(x+y) \frac{dy}{dx} = y \cos(xy) + x \frac{dy}{dx} \cos xy$$

$$\left(\sec^2(x+y) - x \cos xy\right) \frac{dy}{dx} = y \cos xy - \sec^2(x+y)$$

$$\frac{dy}{dx} = \frac{y \cos xy - \sec^2(x+y)}{\sec^2(x+y) - x \cos xy}$$

$$= \frac{-\sqrt{\pi} \cos(-\pi) - \sec^2(0)}{\sec^2(0) - \sqrt{\pi} \cos(-\pi)}$$

$$= \frac{\sqrt{\pi} - 1}{1 + \sqrt{\pi}}$$

$$y = mx + c$$

$$-\sqrt{\pi} = \left(\frac{\sqrt{\pi}-1}{1+\sqrt{\pi}}\right)\sqrt{\pi} + c$$

$$-\sqrt{\pi} = \frac{\pi - \sqrt{\pi}}{1 + \sqrt{\pi}} + c$$

$$\sqrt{\pi} - \left(\frac{\pi - \sqrt{\pi}}{1 + \sqrt{\pi}}\right) = c$$

$$\frac{-\sqrt{\pi}(1 + \sqrt{\pi}) - (\pi - \sqrt{\pi})}{1 + \sqrt{\pi}} = c$$

$$\frac{-\sqrt{\pi} - \pi - \pi + \sqrt{\pi}}{1 + \sqrt{\pi}} = c$$

$$\frac{-2\pi}{1 + \sqrt{\pi}} = c$$

$$y = \frac{\sqrt{\pi}-1}{1+\sqrt{\pi}}x - \frac{2\pi}{1+\sqrt{\pi}}$$

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DO NOT WRITE IN THIS AREA

[10 marks]

Total 20 marks

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SECTION B

Module 2

Answer this question.

2. (a) A geometric progression is such that $S_{\infty} = 9$ and $S_2 = 5$. Given that the common ratio is greater than 0, calculate S_1 of the series.

$$S_{\infty} = \frac{a}{1-r}$$

$$a = \frac{a}{1-r}$$

$$a - ar = a \quad (1)$$

$$S_2 = 5$$

$$a + ar = 5$$

$$a(1+r) = 5$$

$$a = \frac{5}{1+r} \quad (2)$$

$$a - ar = \frac{5}{1+r}$$

$$a(1-r)(1+r) = 5$$

$$a(1-r^2) = 5$$

$$1-r^2 = \frac{5}{a}$$

$$\frac{4}{9} = r^2$$

$$\frac{2}{3} = r$$

$$a = \frac{5}{1+\frac{2}{3}} = 3$$

[10 marks]

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- (b) (i) Use the Intermediate Value Theorem to show that $f(x) = (1+x)e^x - 2$ has a root in the interval $[0, 1]$.

$$f(x) = (1+x)e^x - 2$$

$$f(0) = (1+0)e^0 - 2 = -1$$

$$f(1) = (1+1)e^1 - 2 = 3.44$$

$f(x)$ is continuous on the interval $[0, 1]$
 $f(0) \times f(1) < 0$

By the Intermediate Value Theorem there must be some $c \in [0, 1]$ such that $f(c) = 0$. Therefore there is a root between 0 and 1.

[3 marks]

- (ii) Use three iterations of the Newton-Raphson method with the initial estimate $x_1 = 0$ to approximate the root of f in the interval $[0, 2]$.

$$f(x) = (1+x)e^x - 2$$

$$f'(x) = e^x + (1+x)e^x = e^x(2+x)$$

$$x_2 = 0 - \frac{(1+0)e^0 - 2}{e^0(2+0)} = \frac{1}{2}$$

$$x_3 = 0.38522$$

$$x_4 = 0.3749$$

[5 marks]

GO ON TO THE NEXT PAGE

(c) Determine the coefficient of the term in x^5 in the binomial expansion of $(3x + 4)^9$.

$$\begin{aligned} & (3x+4)^9 \\ & \text{coefficient of } x^5 \\ & \binom{9}{5} (3x)^5 (4)^4 \\ & = 7838208 \end{aligned}$$

[2 marks]

Total 20 marks

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SECTION C

Module 3

Answer this question.

3. (a) A committee of six individuals is to be formed from nine women and three men.

(i) In how many ways can the committee be formed with at least one man included?

$$\begin{array}{l} \text{Men} \quad \text{Women} \\ \underline{3} \quad \quad \underline{9} \\ {}^3C_1 \times {}^9C_5 = 378 \\ {}^3C_2 \times {}^9C_4 = 378 \\ {}^3C_3 \times {}^9C_3 = \frac{84}{840} \end{array}$$

$$\begin{array}{l} \text{Total number of committees} \\ {}^{12}C_6 = 924 \\ \text{Committees with no men} \\ {}^9C_6 = 84 \\ \text{committees with at least one} \\ \text{man} \\ 924 - 84 = 840 \end{array}$$

[4 marks]

(ii) What is the probability that the committee will include five women?

$$\frac{{}^3C_1 \times {}^9C_5}{{}^{12}C_6} = \frac{378}{924} = \frac{9}{22}$$

[4 marks]

GO ON TO THE NEXT PAGE

- (b) How many arrangements can be made using the letters of the word
LEATHERETTE?

$$\frac{11!}{4!3!} = 277200$$

[3 marks]

- (c) Find the general solution of the differential equation $4y'' + 5y' + y = e^x$.

$$\begin{aligned} 4u^2 + 5u + 1 &= 0 \\ (4u+1)(u+1) &= 0 \\ u &= -\frac{1}{4} \quad u = -1 \\ y &= A e^{-\frac{1}{4}x} + B e^{-x} \\ \text{Let } y &= C e^x \\ y' &= C e^x \\ y'' &= C e^x \\ 4C e^x + 5C e^x + C e^x &= e^x \\ 10C e^x &= e^x \\ C &= \frac{1}{10} \\ y &= A e^{-\frac{1}{4}x} + B e^{-x} + \frac{1}{10} e^x \end{aligned}$$

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[9 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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CANDIDATE'S RECEIPT

INSTRUCTIONS TO CANDIDATE:

1. **Fill in all the information requested clearly in capital letters.**

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SUBJECT: PURE MATHEMATICS – UNIT 2 – Paper 032

PROFICIENCY: ADVANCED

REGISTRATION NUMBER:

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FULL NAME: _____
(BLOCK LETTERS)

Signature: _____

Date: _____

2. **Ensure that this slip is detached by the Supervisor or Invigilator and given to you when you hand in this booklet.**
3. **Keep it in a safe place until you have received your results.**
-

INSTRUCTION TO SUPERVISOR/INVIGILATOR:

Sign the declaration below, detach this slip and hand it to the candidate as his/her receipt for this booklet collected by you.

I hereby acknowledge receipt of the candidate's booklet for the examination stated above.

Signature: _____
Supervisor/Invigilator

Date: _____